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A

NEW AND GENERAL INTRODUCTION TO PRACTICAL ASTRONOMY:

WITH ITS APPLICATION TO

GEOGRAPHY, in describing the Earth. TOPOGRAPHY and HYDROGRAPHY, in describing Places on Land and at Sea. HOROMETRY, or the Measurement of Time. The Trial of TIME-KEEPERS. The Obliquity of the ECLIPTIC. The MAGNETIC VARIATION, and Variation Charts of the OCEAN. REFRACTION and PARALLAX. The Horizons of the SPHEROIDAL EARTH. SURVEYING the COASTS, and correcting the CHARTS. Observations of LUNAR ECLIPSES, and JUPITER'S SATELLITES. The Construction of temporary and other Instruments; with Tables of the SUN and FIXED STARS, and a Variety of interesting Copper Plates.

The whole being designed,

As a Course of plain and easy Instructions and Operations, preparatory to the Discovery of

THE LONGITUDE.

BY

SAMUEL DUNN,

Teacher of the Mathematical and Philosophical Sciences, LONDON.

LONDON,

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MDCCLXXIV.

TO
THE HONOURABLE
THE COURT OF DIRECTORS
OF
THF UNITED COMPANY
OF
MERCHANTS OF ENGLAND
TRADING TO
THE EAST INDIES:

THIS WORK,
PECULIARLY DESIGNED
FOR THE IMPROVEMENT
OF
GEOGRAPHY AND NAVIGATION;
IS,
WITH THE GREATEST DEFERENCE
AND RESPECT,
MOST HUMBLY INSCRIBED;
BY THE AUTHOR,

LONDON,
28 October, 1774.

S. DUNN.

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INTRODUCTORY PREFACE.

THE antient aſtronomers, being ſcarce acquainted with the conſtruction of aſtronomical inſtruments, could make but little more than conjectures concerning the ſyſtem of the world. And therefore, from the time of PYTHAGORAS 512 years before Chriſt to that of COPERNICUS 1530 years after Chriſt, the more rational ſuppoſitions concerning the planetary orbs were ſuppreſt, and notions were propagated in their ſtead, which abounded with the greateſt abſurdities.

After COPERNICUS, the illuſtrious and noble TYCHO BRAHE laboured at aſtronomical obſervations; and the ſagacious KEPLER drew his concluſions concerning the inequalities of the planets motions, and the law that regulates their periods round the ſun.

The great philoſopher of this iſland, SIR ISAAC NEWTON, a man who ſeems to have been born and deſigned for enlightening the world in matters of philoſophy, was cotemporary with our countryman Dr. EDMUND HALLEY, who, in the year 1677, formed a plan for inveſtigating the ſun's diſtance from the earth, by the tranſit of Venus over the ſun in 1761.

The aſtronomers of Sweden were remarkably aſſiduous in the making of that obſervation, as was a gentleman in India. Theſe obſerved, that when the planet was near the limb of the ſun, it was not of a circular form. The ſame was obſerved by me near London; and by no other perſon

in England, that I have heard of. The accounts are in Phil. Tranſ. for that year.

In 1762, I endeavoured to find out the cauſe of the phænomenon, as related article 98 of this work; and determined it, with other things relative to light.

By the experiment, I concluded, that, in the ſeparation of ſuch ſhadows from one another, the velocity is much greater than their uſual apparent motion: and, not being unacquainted with the advantages derived from teleſcopes in making aſtronomical obſervations, it appeared to me, that this property might be advantageouſly applied in practical aſtronomy.

The next year, 1763, I made many experiments concerning the vaniſhing and appearing of the ſun, moon, and ſtars, near the ſharp edges of ſeveral kinds of ſubſtances, by direct viſion, as of wood, ivory, braſs, iron, &c.; and at different diſtances from the eye. Theſe were ſucceeded by obſervations of the tranſits of the ſun, moon, and ſtars, acroſs threads ſtretched perpendicularly near the plane of the meridian; from which experiments, I concluded it very eaſy to ſubſtitute an apparatus for a tranſit inſtrument, where the greateſt accuracy is not wanted; and reſerved it to a proper time for publication.

In 1767, I perfected this method in my own apartment; and near the end of the year 1771, being at the houſe of a very learned and ingenious friend in the country, who was provided with every kind of aſtronomical inſtrument, I put up this apparatus for taking meridian tranſits, and offered to be anſwerable for its performing to two ſeconds of time; and it performed accordingly.

Were theſe proofs concerning the invention inſufficient, I could produce many others. And therefore it is obvious, that, if any one has publiſhed any thing of this kind within ſeven years paſt, they have learnt it from my invention; as within that time it has been ſhewn by me to many ingenious perſons, and by this time is communicated to very diſtant parts of the world.

This method of making obſervations has likewiſe, with all poſſible reſpect, been laid before the Board of Longitude;

tude; but no reward has been received for it. And as that manuſcript was too conciſe to contain the application of the method in its proper extent; whatever that was defective in, may be ſeen more at large in this treatiſe.

The names and authorities which have been quoted in the writing of this work, are the greateſt that have ever graced the aſtronomical ſcience. The fixed ſtars were originally obſerved by the ingenious Abbé de la Caille. I have carried them forward to the year 1780. In comparing the obſervations for the obliquity of the ecliptic, I have taken the original obſervations as they ſtand in the authors themſelves, or the literary memoires of the countries to which they belong. And the reſults come out very different from what moſt of thoſe authors have made them.

As it is one principal part of the deſign of this work, to promote the method of obſerving the magnetic variation; I have fully ſet its diſadvantages and advantages before the reader; and have already made proper large delineations for the reception of the variation lines, which may have been well obſerved, at any places of the lands or ſeas.

As the method of finding the longitude at ſea depends much on a good watch; I have ſhewn a great variety of methods of trying ſuch machines, either before they are uſed, or when they are engaged in ſervice.

As to what I have here written, concerning the effects of the figure of the earth in the making of aſtronomical obſervations, it is a path which none has trod before me.

As this ſubject naturally points out a correction of the latitudes and longitudes of places for the purpoſes of geography, and of the coaſts for the purpoſes of navigation; I have ſhewn at large how both of theſe corrections may be eaſily and effectually made: and have given the method of finding the effects of refraction and parallax, which has been known to me at leaſt for ſeven years paſt, whilſt the public have had many operoſe methods laid before them.

When I took the pen to write on this ſubject, it ſeemed as though the contents might have been delivered in two or three ſheets; but having gone a little way, the ſubject, although new, threw ſo many materials in my way, that

that I have been much ſtraitened in comprizing it within five times that number; and, after all, have been under the neceſſity of reſerving the application of the nicer aſtronomical inſtruments to ſome farther publication.

The public have an undoubted right to beſtow cenſure or applauſe. I have written the treatiſe with a deſign to ſhew the eaſieſt and moſt effectual method of determining, 1ſt, The variation of the magnetic needle, in order to obtain exact variation charts of the lands and ſeas; 2d, The gaining or loſing of time-keepers; 3d, The longitudes of places both on land and at ſea. And theſe ſubjects could not well be gone through, without the introduction of other things, which are not here omitted.

The candid will not ſeverely cenſure what hath a tendency to public utility, and is fairly laid before them. But men of degenerate principles may be found, who are ready on many occaſions to ſlight and contemn good performances. Almoſt all the linear plates have been done by my own hands; and they are placed in the work, chiefly near the articles which treat of them.

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A NEW

A NEW and GENERAL INTRODUCTION TO Practical Aſtronomy, &c.

1. THE Science of Aſtronomy is of ſuch antiquity and utility, that to attempt to write any thing in favour of it, would be ſomewhat like an attempt to add luſtre to a diamond; this ſcience having been generally allowed, for a ſeries of ages, by men of the greateſt wiſdom and underſtanding, to be one of the nobleſt and moſt extenſive, that ever exerciſed the human mind.

2. If we may give any credit to the hiſtory of antient times, almoſt all the learned and great men amongſt the antients were admirers and promoters of this ſcience. And the utility of aſtronomy is ſuch, in the ſtudy and practice of modern affairs, that it is greatly aſſiſting in navigation and foreign commerce, ſupplies the imperfections which ariſe in clock-making and other horological arts, and is happily inſtrumental in explicating and demonſtrating ſome of the moſt intricate phænomena of nature.

3. Hence it follows, that we have a long liſt of Aſtronomers from the earlieſt accounts of time: amongſt whom were, Adam, the firſt of the humane race; Seth, his ſon; Enoch, the ſeventh from Adam; Ham, the ſon of Noah; and Nimrod, the firſt king who reigned in Aſſyria, at the time when the tower of Babel was built. Belus, the Aſſyrian, in whoſe time the tower of Babel is ſaid to have been uſed

by the Chaldeans for obſerving the riſing and ſetting of the celeſtial bodies; Abraham, the Patriarch; Moſes, the Jewiſh legiſlator; Atlas, king of Mauritania; Solomon, king of Iſrael; Atreus, king of Argos in Greece, in whoſe days the famous Argonautic expedition was executed; Numa Pompilius, ſecond king of the Romans; Thales of Miletum, and Anaximander his kinſman; Pythagoras, of Samos; Meto, the Athenian, inventor of the golden number, wherein the ſun and moon are ſuppoſed to make all their various poſitions to each other in 19 years; Plato, the divine Athenian; Ariſtotle; Archimedes; Hipparchus; and Julius Cæſar, firſt emperor of the Romans, who reformed the Roman year, and reduced it to 365 days and 6 hours, which is nearly the account now made uſe of throughout all the civilized part of the world. Theſe were before the beginning of the Chriſtian æra.

4. Since the coming of Chriſt, Tiberius Cæſar; Seneca the philoſopher, who wrote of comets and agreeable with modern diſcoveries; Pliny, the naturaliſt; Plutarch, the hiſtorian; Menelaus, the poet; Claudius Ptolemy, of Peluſium in Egypt, a great reformer of geography; Pappus, of Alexandria; Charlemaigne, king of France, and emperor; Albategnius, the Syrian; Alphonſus, the 10th king of Caſtile and Leon; Paracelſus; Leo, emperor of Conſtantinople; Richard the Second, king of England; Chaucer, the poet; Ulugh Beig, nephew to the great Tamerlane; Regiomontanus, and Bernard Walther his diſciple; Chriſtopher Columbus, diſcover of America or the New World; Vernerus, of Nuremberg, diſcoverer of the method of ſubdividing degrees into minutes, and minutes into ſeconds, commonly called Nonius's diviſion; Nicolas Copernicus, reviver of the true ſyſtem of the world, which was firſt thought of by Pythagoras, and afterward demonſtrated by Sir Iſaac Newton; Gerard Mercator; the Noble Tycho Brahe; William landtgrave of Heſs; Scaliger; Grotius; Bayer, who put the Greek letters to the aſteriſms; Lord Naper, inventor of the logarithms; Galilæus; Kepler, firſt diſcoverer of the elliptical orbits of the planets; Edward Wright of London, firſt inventor of the true ſea chart aſcribed to Mercator; Biſhop Wilkins, one of the firſt members of the royal ſociety of London; Sir Jonas Moore; Gaſſendus; Des Cartes; Sir Chriſtopher Wren; Mr. Flamſted;

ſted; Sir Iſaac Newton; Dr. Edmund Halley; and many others.

5. Hence it appears that aſtronomy has been ſtudied and cultivated by the moſt ancient people of the world, and during the monarchies of the Aſſyrians, Egyptians, Grecians, and Romans. In the leſs civilized parts of the world, as it has been, ſo it continues to be, looked on as almoſt a celeſtial ſcience. The preſent monarchs of the ſeveral parts of the earth countenance this ſcience, with a two-fold view; firſt, as it exerciſeth the mind concerning objects which are truly great, the motions, diſtances, and appearances of the celeſtial bodies, which are aſtoniſhing to contemplate; and ſecondly, as it tends naturally to expunge all errors that may ariſe in geography and navigation; for it is the natural tendency of this ſcience to bring geography to its greateſt perfection, and to enable mariners to determine the place of a ſhip at ſea, more exactly than by any other ſcience whatſoever.

6. In a courſe of general education, adapted for extenſive purpoſes of life, this ſcience is of no little importance. By a diligent and inoffenſive exerciſe of the ſenſes, it cannot but thereby be the means of improving them, and make them more apt for diſcernment, on many occaſions which occur in buſineſs and life. But the utility of aſtronomy is ſuch, that in carrying on commerce with any diſtant parts, which are ſeparated by the ocean, no ſafety can be expected, when the ſhip is bewildered through ſtorms or tempeſts in croſſing the pathleſs ocean, without the practice of aſtronomy.

7. Were there no other advantages to be derived from this ſcience, they would be exceedingly great, becauſe, by the extenſion of commerce, and the carrying on of trade from one diſtant part to another, the conveniencies of life are ſupplied, the artiſans of different nations at great diſtances from each other are kept conſtantly at work, the alarms of war and domeſtic tumults are hereby avoided, and finally this is one of the beſt and moſt laſting methods of enriching a nation, and making its individuals happy.

8. But practical aſtronomy is of the greateſt utility in aſcertaining the lengths of either greater or leſs periods of time. The practical aſtronomer can, by the help of a very few data, determine the length of ſuch periods as would not only be very tedious and difficult to keep account of,

from one day to another, but alſo determine the errors which ariſe in the going of an automaton, to the greateſt nicety diſcoverable by the ſenſes, which errors, otherwiſe, would perhaps be hardly ever diſcovered by any human contrivance. And by this ſcience it is that the irregular apparent motions of the celeſtial bodies are diſcovered; the whole ſcience of dialing is performed; the errors of ſun-dials are diſcovered, and proper allowances made for thoſe errors; and finally, hereby the true merit of every horological inſtrument-maker is eaſily and certainly known; and the merit of the moſt ingenious horological artiſt may be readily examined by himſelf, in order to make corrections or farther diſcoveries.

9. The exceeding rapidity with which light paſſeth through the heavens; the laws of nature, whereby the ſyſtem of the world is ſo harmoniouſly continued; the idea which we are able to form concerning the more remote parts of the univerſe; and many other concluſions, equally engaging and ſurprizing to the inquiſitive mind, are but the natural productions of practical aſtronomy, without which thoſe truths muſt have for ever lain hid and undiſcovered by human nature.

10. The word aſtronomy is formed from the Greek words ἀςὴρ (*aſter*) a ſtar, and νόμος (*nomos*) a law or rule. And the ſcience of aſtronomy (ἀςρονομία, or *aſtronomia*), is that of the ſun, moon, planets, and fixed ſtars.

11. Aſtronomy may be conſidered, as conſiſting of two principal parts, phyſical and practical. Phyſical Aſtronomy points out and demonſtrates the cauſes of the irregularities which ariſe in the motions and appearances of the celeſtial bodies; and Practical Aſtronomy is that whereby the places of the ſun, moon, planets, and fixed ſtars, in the heavens, are obſerved, with their other phænomena, by the help of proper aſtronomical inſtruments.

12. The principal inſtruments uſed in practical aſtronomy are, firſt, angular inſtruments, for taking the poſitions of the celeſtial bodies to each other, and to obſervers on the earth's ſurface, or for taking their poſitions to certain circles and points in the heavens, and on the earth. Or, ſecondly, when ſuch angular poſitions or diſtances cannot be well obſerved, the practical aſtronomer may be greatly aſſiſted by an accurate clock, or other timekeeper, that can

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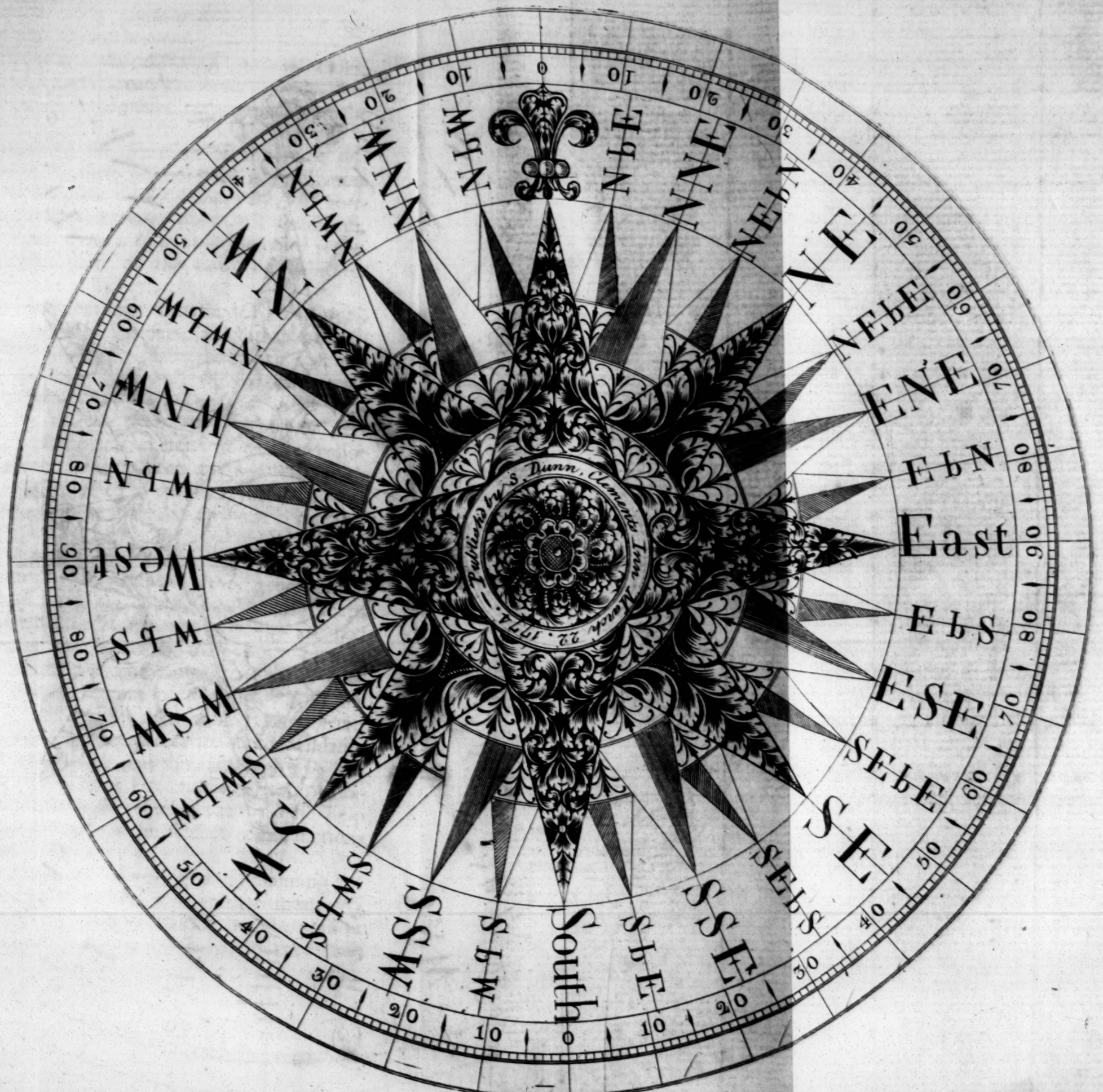
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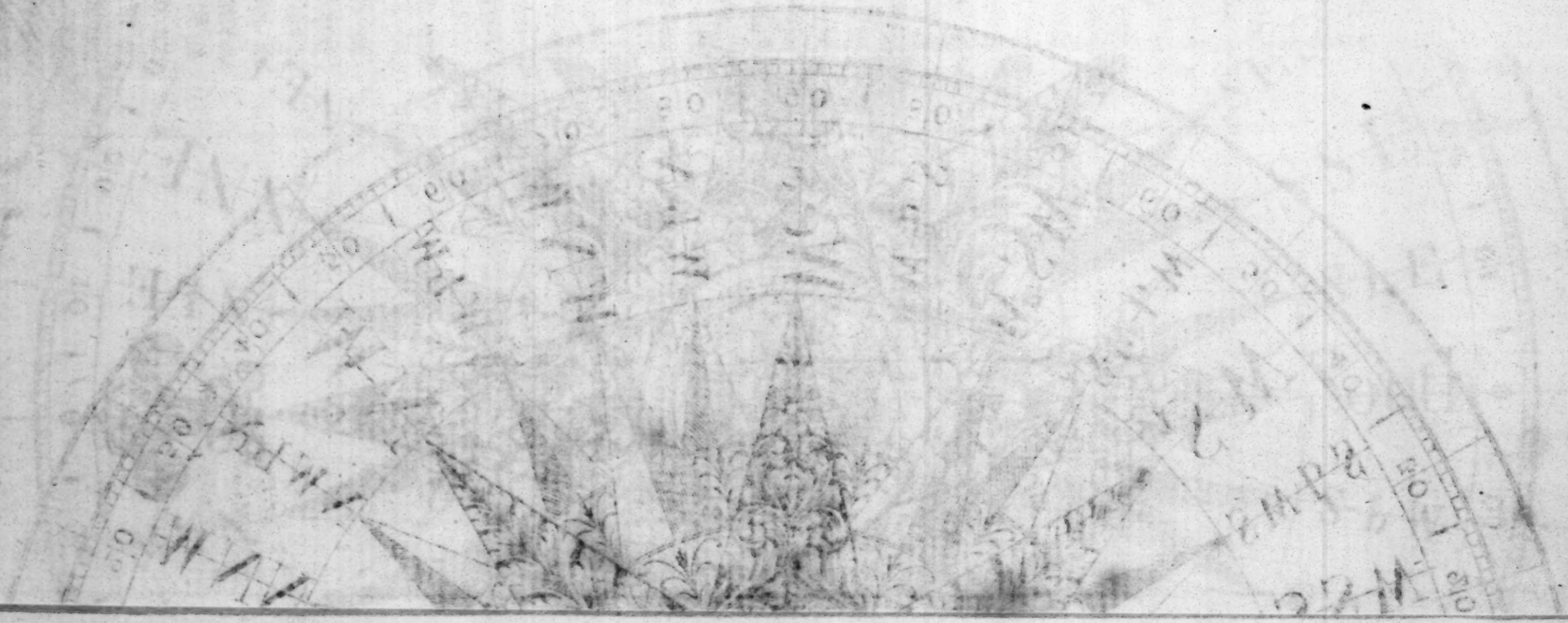


be made to keep time nearly conformable with the apparent diurnal revolution of the heavens.

13. Topography deſcribes the different parts of cities, towns, and villages, pariſhes, manors, woods, fields, &c. as they are ſituated amongſt themſelves at any places of the earth.

14. Geography deſcribes the ſituation of the moſt diſtant places on the earth, by their poſitions to, and diſtances from, each other, however great thoſe diſtances are, and however lands or ſeas do interpoſe between them. This ſcience, in its moſt perfect ſtate, gives a true and adequate repreſentation of the various parts of the earth or habitable world.

15. Hydrography deſcribes the ſituation of the ſeveral ſands, ſhoals, rocks, eddies, currents, and other phænomena which are to be met with, throughout the vaſt extended oceans, and near to, or remote from, the ſea-ſhores.

16. Navigation is that ſcience whereby the mariner conducts a ſhip through the ocean, from one country or port to another country or port; and whereby he can determine at any time, although the ſhip has been any how driven by the winds or waters, what place in the ocean ſhe is in, and what courſe and diſtance ſhe muſt ſail, to arrive at her deſired port.

17. Surveying is that ſcience whereby accurate delineations are made, either in topographical plans, geographical maps, hydrographical charts, the delineations of the maps of countries, or any parts of them; and whereby are had accurate delineations of the coaſts of the ocean. Hence, topography being extended becomes geography, as geography in its leſſer extent becomes topography. And the ſciences of topography, geography, hydrography, and navigation, have each of them a near affinity to, and dependance on, the principles of ſurveying; and this, although in a leſs extent it may be performed by a few mathematical principles, yet in a greater extent, it requires the greateſt help and aſſiſtance from aſtronomy, or the ſcience of the motions of the celeſtial bodies.

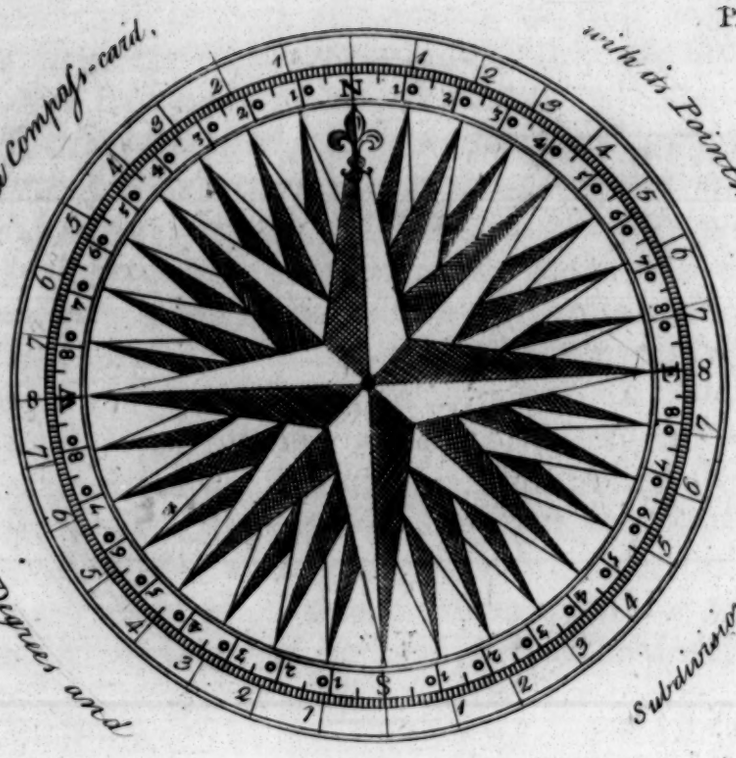
18. The uſual method of ſurveying and planning of lands which are of no great extent, is by either meaſuring the lines on the lands, and thereby making out the plan or map; or, when the lines cannot be meaſured, to take ſuch angles

angles as the practice will permit to be taken; and by these, with certain lines that can be measured, to make out the plan or map. But this method will not hold good for plans and maps which represent a very large extent of country, because, when the distances become great, the positions of places to each other cannot be seen; and by carrying on the survey by measuring of short distances and taking many angles, the greater parts of which are erroneous in some degree, the survey becomes at last very imperfect, if not altogether unfit to be depended on. And hence it is that most geographical delineations are imperfect, unless they are regulated and adjusted by astronomical observations.

19. The same kind of errors arise when the mariner has conducted his ship through the ocean by the usual method of sailing. In this practice he measures the distance sailed, and the course steered; both of which being subject to no small errors from day to day, until a great number of those errors are introduced, he at length arrives at a place in the ocean, where he does not expect himself by his reckoning; and therefore stands in need of some method to correct that error, and such a method is only to be had from practical astronomy.

20. The first principles on which the practice of astronomy is founded, are, the figure of the earth; the direction of gravity on all parts of the earth's surface; the position of the visible horizon; the apparent revolution of the celestial bodies round the axis of the earth, or that axis produced into the heavens, once in twenty four hours nearly; the formation of the meridians and parallels of latitude on the surface of the earth and sea; the formation of the meridians, parallels of declination, and parallels of latitude in the heavens; the construction of the earth's equinoctial line, the equator and ecliptic in the heavens; the great circles on which the distances of places on the earth's surface are measured; the angles which those great circles make with their respective meridians, by which their bearings or positions are estimated; the construction of the great circles in the heavens, by which the distances of the fixed stars from one another are reckoned, and the positions of those great circles to one another and to other circles of the heavens, which give names to various properties of the sphere; these and some others have their foundation in the physical

The Compass-card, with its Points, Degrees and Subdivisions.



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phyſical part of aſtronomy, being clearly deduced from the properties of matter and motion.

21. The principal buſineſs of the practical aſtronomer is, to make and regiſter his obſervations, without either accounting for, or enquiring into, the cauſe of their phænomena. In the performance of this, it will ſometimes be requiſite for him to underſtand the uſe of the tables that are applicable in the practice of aſtronomy, and the making of ſuch calculations as are aſſiſting in the practice; and it may ſometimes prevent him from making erroneous ſuppoſitions, if he underſtands the principles on which the calculations are founded.

22. Hence the practical aſtronomer may be conſidered as one or other of the three following claſſes; namely, firſt, the mere obſerver, who is able to make the obſervations and regiſter them regularly and faithfully as they occur, in whom there is but little elſe required than common fidelity, judgement, and care; ſecondly, the obſerver who can make his obſervations and calculate by the tables, without knowing the reaſons on which the calculations and tables are conſtructed; and thirdly, the obſerver who underſtands the phyſical cauſes on which the tables are formed, the manner of calculating by them, the making of the obſervations, the conſtruction of the inſtruments, the compariſon of the obſervations with the predictions, and thereby the correction of the tables for the uſe of future times, and the improvement of geography and navigation.

23. The inſtruments commonly uſed in practical aſtronomy may be conſidered as of two principal kinds, thoſe which are moſt ſimple and portable, and thoſe which are complex, large, and not eaſily to be removed from one place to another, and therefore the latter are generally fixed up or placed in aſtronomical obſervatories, whilſt thoſe of the former kind may be readily removed from one place to another, and uſed as opportunity permits. The former kind of inſtruments, namely, the more portable, may be conſidered as of two different claſſes, one of them applicable only when the inſtruments themſelves are either fixed or in a ſtate of reſt, and the other claſs eaſily to be applied by an obſerver in a portable manner, whether he be on land or at ſea, notwithſtanding the motion of his body, or the motion of a ſhip.

24. To

24. To the firſt kind of theſe inſtruments, the more large and accurate ones in the fixed obſervatories, geography and navigation are indebted for thoſe accurate tables which have been formed from one age to another, concerning the motions of the celeſtial bodies, and their ſituations in the heavens after any periods of time. Sometimes the apparent interpoſition of the celeſtial bodies with one another have been obſerved by the naked eye, whereby the obſervations may be ſaid to have been made, as it were, with inſtruments as large as the heavens themſelves, in which there could have been no errors but ſuch as may be ſuppoſed to have ariſen from a defect of the ſenſes; ſuch as occultations of the fixed ſtars by the moon, and both lunar and ſolar eclipſes, obſerved by the naked eye unaſſiſted by optical glaſſes. And many ſuch obſervations have been made in different parts, which have conduced toward correcting the aſtronomical tables. To the latter or more portable claſs of thoſe inſtruments, they owe the numerous obſervations which have been made by voyagers and travellers, who have ſo amply contributed toward ſettling the ſituation of places, and the formation of accurate maps and charts, of the various parts of the known world.

25. The more portable inſtruments may be conſidered farther as one of the two following kinds; either as being formed of wood, ſimply from mathematical principles; or compounded and complex by a mixture of ſcience and mechanical arts. The former of theſe kinds have been chiefly in uſe till within about a century laſt paſt. The latter have been very much improved ſince that time, by the refinements made in framing, dividing, and engraving, and the additions that have been made to them, by help of late optical diſcoveries, ſuch as the adapting of teleſcopes and optic glaſſes to mathematical inſtruments.

26. In the uſe of aſtronomical inſtruments daily experience proves, that their parts are in a continual ſtate of change and alteration. Therefore, ſuch inſtruments as are fixed are frequently requiring new adjuſtments, before they can be applied with the deſired accuracy; and ſuch as are moſt portable are ſo affected by the change of the ſtate of the air and weather, as to require an adjuſtment of their parts, previous to almoſt every new obſervation.

27. Theſe imperfections will ever attend inſtruments of either the ſmaller or larger kind, even when they are made of braſs, except where the greateſt mechanical artifice is uſed to counteract them. For the beſt angular inſtruments that are uſed ſeldom exceeding eight or ten feet in radius, and the leſſer inſtruments being frequently not an eighth nor a tenth part of that dimenſion, whilſt the arches of either of theſe inſtruments repreſent either arches of the earth or of the heavens, the ſmalleſt error imaginable in the framing or dividing of the inſtrument, or the alteration in its parts by the change of the air or weather, muſt neceſſarily anſwer to a conſiderable error in the arch which it repreſents, either of the earth, or of the heavens.

28. For theſe reaſons the antient aſtronomers wiſely preferred large inſtruments, in the making of their beſt aſtronomical obſervations; and the modern aſtronomers either ſet up marks, or make uſe of objects at a conſiderable diſtance, whereby to adjuſt ſome of their inſtruments, when the parts of thoſe inſtruments, amongſt themſelves, cannot be made to contribute to any ſuch rectification.

29. In order to the better underſtanding of the practice of aſtronomy, it is requiſite to be acquainted with the following definitions and properties of the ſphere.

30. A ſphere or globe is a body perfectly round, every part of its ſurface being the ſame diſtance from the centre, or point, in the middle of the ſphere or globe. A great circle of a ſphere or globe, is the largeſt that can be drawn on its ſurface, and is ſuppoſedly divided in 360 equal parts called degrees, each degree into 60 equal parts called minutes, and each minute into 60 equal parts called ſeconds, &c. In the practice of aſtronomy, it is uſual to write down the degrees, minutes, and ſeconds, at full length without any abbreviation, but after the ſeconds, inſtead of thirds, aſtronomers uſually write down tenths of a ſecond.

31. An hour of time is divided into 60 equal parts, called minutes of time, and a minute of time is divided into 60 equal parts, called ſeconds of time. After the ſeconds of time, it is uſual to expreſs any overplus by tenths of a ſecond of time, but ſome aſtronomers retain the old method of expreſſing any overplus by ſixtieths of a ſecond of time, which are called thirds of time.

32. Aſtronomical inſtruments, that are made uſe of for taking the angular diſtances of the celeſtial bodies, are uſually divided and ſubdivided into degrees and minutes of a degree. If they are large enough, the ſubdiviſions are carried to ſeconds. When the inſtrument is fixed, and the application of the celeſtial body to it, by means of the apparent revolution of the heavens, is to be obſerved, the obſerver makes uſe of a clock, which is generally made to beat ſeconds of time; and the clock being fixed to a firm wall, or other place, the inſtant of time when the celeſtial body appears to touch the inſtrument, or the ſilver wire in its teleſcope, if it has any ſuch, this is noted by the obſerver, inſtead of the other method of practice, for taking the real angle.

33. The perfection of this latter kind of inſtrument, ſuch as pendulum-clocks, depends almoſt wholly on their being fixed to a firm building. Add hereto that a good pendulum-clock is found by experiments to keep time with great regularity and exactneſs, when it is clean, in good order, and well adjuſted.

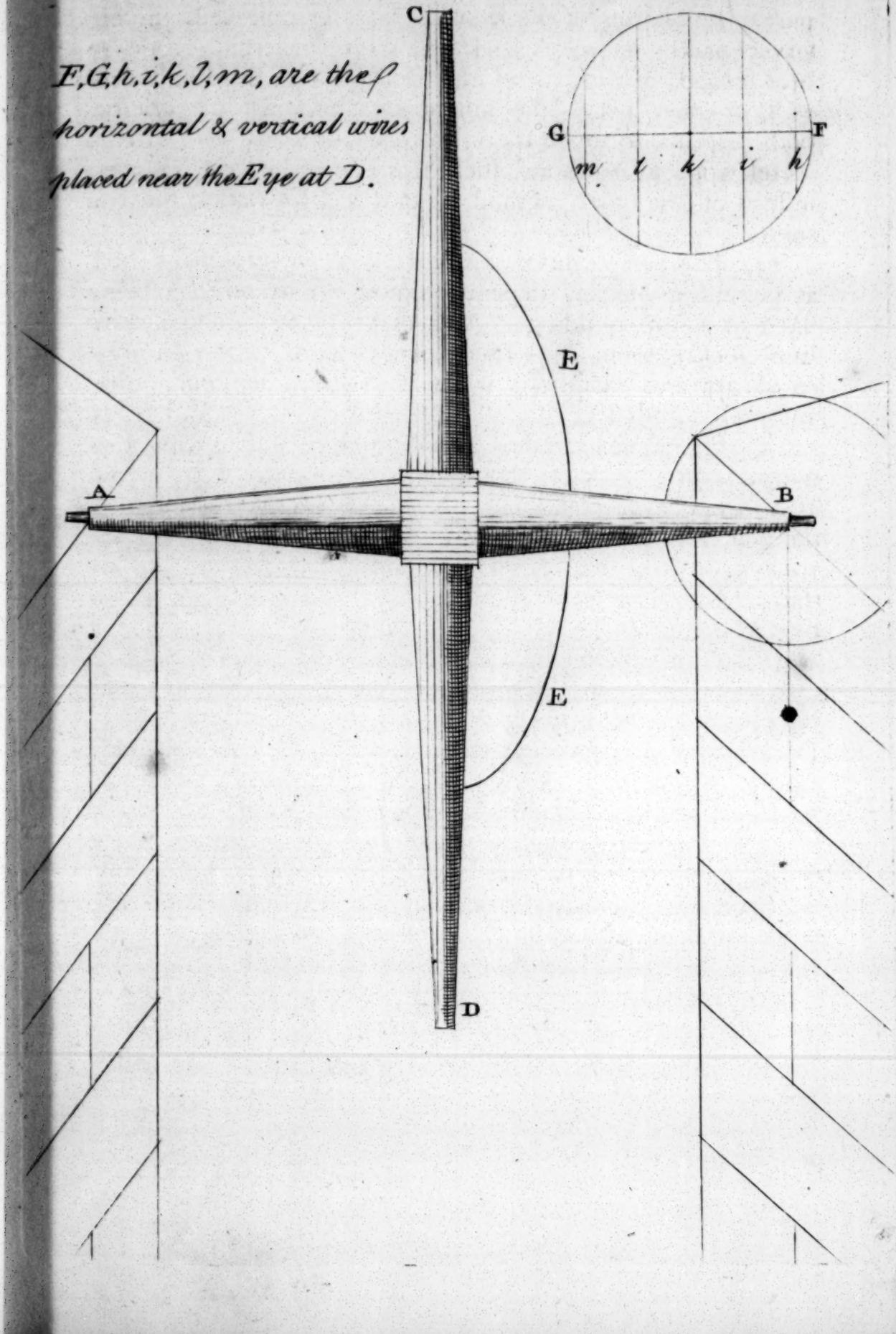
34. The moſt capital of all aſtronomical inſtruments, that is uſed with a clock, is the tranſit inſtrument. It is placed as accurately as poſſible, in the plane of the meridian of the place of obſervation; and by moving round an horizontal axis, the ſun, moon, and ſtars, are obſerved, when they come to the inſtrument, and the time noted, as ſhewn by the clock.

35. This inſtrument was invented by the celebrated Orlaus Roemer, who, according to the Miſcel. Berolin. pars II. p. 276, ſent an account thereof from Copenhagen, December 15, 1700, to M. Leibnitz, in the following words: "Out of a number of thoughts, I ſhall at preſent "collect one concerning an inſtrument, for which I have "longed to ſee a proper chamber or building theſe 25 years, "but never met with one compleatly to my wiſhes. The "inſtrument is a meridional axis ſix feet long, revolving "round the poles A B, ſupported by ſolid walls, and car-"rying in its middle the meridional circle E of 4 or 5 "feet diameter, with the tube C D of 6 or 8 feet, which, "revolving by the axis A B, purſues the whole meridian "to its very horizons, within 4 or 5 degrees. The in-"ſtrument muſt be fixed in a covered apartment 6 feet

"broad,

The Transit Instrument.

F, G, h, i, k, l, m, are the horizontal & vertical wires placed near the Eye at D.



G.R. III.
BRITISH MUSEUM

" broad, which is the diſtance of the walls. Its length, " according to the meridian, ſhould be 30, or at leaſt 25 " feet, and its height not leſs than 20; and that the whole " meridian may appear on the circumvolution of the tube, " the top of the walls ſhould have a continued fiſſure or " chink 4 inches wide, the walls of this nearly biſected " apartment being bound together by the intervention of " a few iron bars. The inſtrument itſelf being eaſily con- " trived, the chief difficulty lies in procuring a building of " proper ſtrength and ſituation for the admiſſion of it, " which muſt be conſidered as its neceſſary appendage and " accompliſhment. It can be no ways neceſſary to make " either the inſtrument or its receptacle any larger; beſides " three clocks of the ſame ſize and fabric, there is no oc- " caſion for any other inſtrument; all our care ſhould be " confined to the firm ſtructure, and conſtant motion of it. " Its uſe conſiſts in determining the right aſcenſions and " declinations of all viſible points in the heavens, the for- " mer by the clocks, the latter by the circle, adhering to " the tube and axis. If it were not over-tedious, I could " annex ſeveral uſeful mechanical obſervations in manage- " ing the parts of this inſtrument, having experienced it " for eight years, during which time I have made uſe of " ſuch an axis (though very imperfectly fixed) with no " contemptible ſucceſs, which, God willing, I may here- " after demonſtrate by publiſhing my obſervations." From this account by the celebrated inventor, the tranſit inſtrument was invented within a century paſt. It is now conſtructed in a manner ſomewhat different from that deſcribed by M. Roemer, and on account of its utility is frequently introduced as an additional part to ſeveral other aſtronomical inſtruments. The more antient aſtronomers had their fixed quadrants, which they uſed as mural arches; but the difference between the tranſit inſtrument and thoſe, may be conſidered as ariſing from the introduction of an horizontal axis, and a vertical motion round that axis. As the inſtrument is now improved, it is adjuſted horizontally, either by the ſpirit level, or by a plumb line; and when a meridional line, or, which is the ſame thing, a meridional mark is found, and the inſtrument adjuſted to that mark, the office of the inſtrument is to receive the contact of any

of the celeſtial bodies as they do apparently paſs over the meridian of the place of obſervation.

36. The beſt braſs angular inſtruments, that are now uſed in fixed obſervatories, are about eight feet radius or ſemidiameter. In theſe inſtruments the minutes of a degree are very conſpicuous; but for rendering the ſeconds legible, there have been various contrivances, amongſt which, when the various imperfections which attend mechaniſm have been duly conſidered, I take the ſubdiviſion by a vernier, or nonius diviſion, to be the moſt unexceptionable and beſt.

37. In ſuch an inſtrument, when the aſtronomer has made his obſervation by it, and comes to read the ſubdiviſions of the inſtrument, to ſeconds of a degree, however exactly the inſtrument is made and ſubdivided, he will find it very difficult to read to a ſecond of a degree, and more difficult to read to half a ſecond of a degree. And therefore it is no leſs aſtoniſhing than abſurd, that aſtronomers ſhould pretend to give us, as they do in their publications, obſervations and calculations, to the tenth part of a ſecond of a degree; which can never be taken, by the beſt inſtruments.

38. As the celeſtial bodies do appear to riſe, ſet, and paſs through the heavens, and round the earth, once in a day nearly, aſtronomers obſerve the time of their ſouthing, northing, &c. by a clock, and a teleſcope fixed to the inſtrument with which they obſerve. The teleſcope uſually fixed to ſuch angular inſtruments of the beſt kind beforementioned, generally makes the object appear 80 times larger than the naked eye, and conſequently the apparent ſwiftneſs of the ſun, moon, and ſtars, as viewed through ſuch a teleſcope, may be ſaid to be 80 times ſwifter than the apparent ſwiftneſs, as obſerved by the naked eye.

39. Whilſt the clock is beating even ſeconds of time, the aſtronomer endeavours to take the inſtant of the phænomenon he is to obſerve, as accurately as poſſible. But when all his care and attention have been exerted, he cannot declare the inſtant of the phænomenon more accurately than to a fourth or a fifth part of a ſecond of time, which latter anſwers to 3″ of a degree of diſtance in the heavens. Aſtronomers frequently content themſelves with obſerving to a ſecond of time, which is 15″ in the heavens. The

reaſon

reaſon of theſe apparent motions of the celeſtial bodies, from eaſt to weſt, and quite round the heavens, in a day, is as follows.

40. The ſun is placed near the centre of the ſolar ſyſtem, and nearly at reſt, and the earth is carried round the ſun in a year. Whilſt the earth is thus moving round the ſun, the earth's axis retains the ſame parallel poſition, and the earth revolves round that axis once in a day, in a direction from weſt towards the eaſt.

41. The inhabitants of the earth being ſituated on the ſurface of its globe, and the fixed ſtars being at unmeaſurable diſtances from the ſun and earth, the diurnal rotation of the earth, from weſt toward the eaſt, deceives the ſight of ſpectators on the earth's ſurface, and exhibits an appearance of the fixed ſtars moving round the earth's axis, or that axis produced to the heavens, in a direction from eaſt towards the weſt, contrary to the true diurnal motion of the earth; and this is called the apparent or viſible motion of the heavens.

42. The path of the earth in its orbit being between the ſun and the fixed ſtars, the latter appear to gain on the ſun's apparent, or viſible diurnal revolution round the axis of the earth, by an interval of 3 minutes and 56 ſeconds of time nearly.

43. The earth's rotation round its axis is ſuppoſed and found by aſtronomers to be by an equable and uniform motion, and therefore aſtronomers pay no little regard to the kind of time, which is reckoned according to this apparent motion of the ſtars, becauſe it is uniform and equal; and this they call ſiderial time.

44. The earth's motion in its orbit being unequal, at ſome places ſwifter than at others, it follows that no equal time can be had purely from obſerving the apparent diurnal revolution of the ſun round the earth's axis; and this unequal time is called ſolar time, and ſometimes, though improperly, it is called true time.

45. The year is again ſuppoſedly divided and ſubdivided into days, hours, minutes, and ſeconds, of equal time, as meaſured out by a continued ſucceſſion of equal portions or parts of duration; and this time is called equal time, or mean ſolar time.

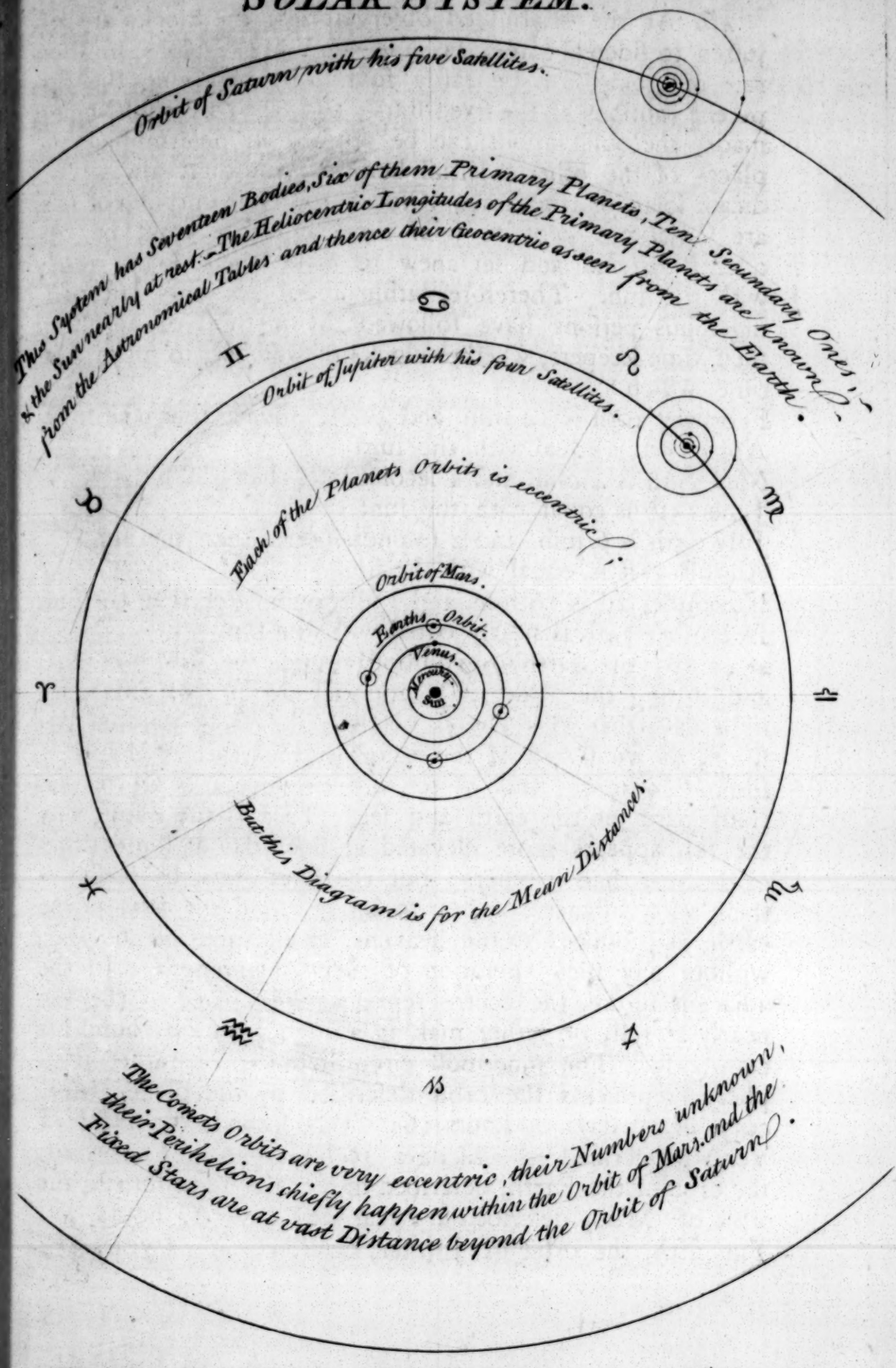
46. At

46. At the eſtabliſhed obſervatories, the clocks are adjuſted to ſidereal time, becauſe the imperfections in their rate of going can be eaſily found, by obſerving the apparent motions of the fixed ſtars; and, the corrections being made, the obſervations can be applied in determining the places of the other celeſtial bodies. Sun-dials always indicate ſolar, or true time. Clocks, for civil purpoſes of life, are ſometimes kept to ſolar time, but they muſt be frequently altered and ſet anew to make them keep nearly with the ſun. Therefore, within about 20 years laſt paſt, ingenious perſons have followed the method of adjuſting their time-keepers, whether clocks or watches, to mean ſolar time, which

February 10th is 14 min. and 41 ſec. ſooner than the ſun.
April 15th is equal with the ſun.
May 15th is 4 min. and 1 ſecond later than the ſun.
June 15th is equal with the ſun.
July 25th is 6 min. and 2 ſeconds ſooner than the ſun.
Auguſt 31ſt is equal with the ſun.
November 2d is 16 min. and 13 ſeconds later than the ſun.
December 24th is nearly equal with the ſun.

47. By the earth's revolution round the ſun in a year, and during that time its rotation round its own axis once in a day, that axis always keeping the ſame parallel poſition, the viciſſitude of the ſeaſons of the year, or of ſpring, ſummer, autumn, and winter, are produced, at all the various parts of the earth and ſea. This is the cauſe why the ſun appears more elevated at noon-day at ſome times of the year than at others: but the fixed ſtars, by reaſon of their vaſt diſtances, appear to move round the axis of the earth, as produced to the heavens, at all times of the year, without any ſuch alteration of their phænomena. In the adjacent figure, the centre repreſents the place of the ſun nearly at reſt, or rather making a ſmall gyration round his own body. The innermoſt circumference, or rather periphery, repreſents the orbit deſcribed by the planet Mercury, in 85 days 23 hours 16′. The ſecond, the orbit of Venus, deſcribed in 224 days 16 hours 49′. The third, the orbit of the Earth, deſcribed in a year. The fourth, the orbit of Mars, deſcribed in 1 year 321 days 23 hours 31′. The fifth, the orbit of Jupiter, deſcribed in 11 years 317 days

SOLAR SYSTEM.



The Sun's Mean Distance from the Earth is near 90 Million Miles.

S. D. Delin.t

days 8 hours 28′. And the ſixth, the orbit of Saturn, deſcribed in 29 years 187 days 20 hours 34′.

48. As theſe different planets do not move round the Sun in the ſame, but in different planes, they muſt neceſſarily appear ſometimes above, and ſometimes below, the plane of the Earth's orbit; this makes what is called the latitudes of thoſe planets; and as their periodic times are different from each other, it follows that they muſt ſometimes appear in one part of the heavens, and ſometimes in another, but always near the plane of the Earth's orbit, which is called the ecliptic. And comparing the diſtances of Mercury and Venus from the Sun, with the theory, the former cannot be more than 28 degrees, nor the latter more than 48 degrees from the Sun.

49. The celeſtial bodies, whenever they appear to ſpectators on the earth or ſea, are apparently above the horizon of the place of obſervation, and that horizon is an extended plane, dividing or ſeparating the upper part of the heavens from the lower; and therefore whenever the celeſtial bodies are not apparently above the horizon of the place of obſervation, they cannot be ſeen, although the ſky be ever ſo clear.

50. The circumference of the viſible horizon is ſuppoſedly divided and ſubdivided into degrees, minutes, and ſeconds, for the purpoſes of aſtronomy and geography; but for the purpoſes of navigation, it is uſually divided and ſubdivided into points, quarter points, and half quarter points of the compaſs, as in the delineation of the compaſs, with its degrees and points annexed.

51. Hence it is, that, ſtrictly ſpeaking, as many different places as there are on the ſurface of the earth, or the ſeas, ſo many different horizons are there, or an horizon peculiar to each of theſe places; and as the places themſelves are coincident with the ſurface of the earth and ſea, and the whole globe of the earth and ſea is continually in rotation, the horizons of places are continually revolving with the places themſelves.

52. Hence it is, farther, that the ſun apparently riſeth towards the eaſt, and apparently ſets towards the weſt, and is at his greateſt elevation at noon-day; but at his greateſt depreſſion below the viſible horizon at midnight; that a point in the viſible horizon, directly under the ſun at noon-

day,

day, is ſaid to be in the meridian of the place of obſervation; and a point in that horizon, directly over the ſun at midnight, is ſaid to be in the meridian of the place of obſervation; and this meridian, or truly north and ſouth ſtraight line, is of the greateſt uſe and utility in the application of aſtronomical inſtruments, and making aſtronomical enquiries.

53. This meridian line may be drawn of any ſmall determinate length, and nearly coincident with the horizon of a place of obſervation, a variety of ways, ſome of which are more correct than others; but the practical aſtronomer may perform this to ſome tolerable degree of accuracy, by attending to and practiſing the following method, which I diſcovered by experiments.

EXPERIMENT.

54. Having delineated ninety concentric circles on a large ſemicircular plate of 29 inches and two thirds of an inch diameter, and divided the arch into degrees, and by help of diagonals ſubdivided it to every five minutes of a degree, I cauſed ſeveral prints to be printed from this plate, after the uſual manner, and took a quadrant of theſe prints, after they were printed and dried; and the meaſure of the radii of the quadrant, which ſtood at right angles to each other, being compared with the radius or ſemidiameter of the plate, were as follows:

	Inches.
Radius of the plate as delineated, — — —	14,83
One radius of the print as printed, — —	14,70
Shrinking of this radius by printing,	,13

	Inches.
Radius of the plate as delineated, — — —	14,83
Other radius of the print as printed, — —	14,65
Shrinking of this radius by printing, — —	,18

So the difference of the two radii of the print was $\frac{5}{100}$ths of an inch.

EXPERI-

EXPERIMENT.

55. Having procured a ſquare deal board of the dimenſions of the quadrant, and an inch thick, with proper croſs bars to prevent it from ſhrinking in the breadth; I paſted this printed quadrant on the board with a good quantity of ſtrong paſte, and, when it was dry, I meaſured the two radii of the quadrant ſtanding at right angles to each other, and comparing them with the radius of the plate, they were as follows:

	Inches.
Radius of the plate as delineated, — —	14,83
One radius after paſting on board, — —	14,78
Error by printing and paſting on board, —	,05

	Inches.
Radius of the plate as delineated, — —	14,83
Other radius after paſting on board, — —	14,76
Error by printing and paſting on board, — —	,07

So the difference of theſe two radii of the print, after paſting, was $\frac{2}{100}$ths of an inch, which was but two fifths of the error introduced by printing only.

56. In the arch of this quadrant, an inch meaſured 3 degrees and 50 minutes nearly; and therefore the diſtorted radii, by printing and paſting, which amounted to $\frac{2}{100}$ths of an inch, anſwered to 5 minutes of a degree nearly.

EXPERIMENT.

57. Seeing the radii of the quadrant were ſo little diſtorted by printing and paſting, without any regard to a correction, I paſted another of theſe printed quadrants; and in the doing of it, by help of a meaſure on a ſlip of paper, corrected the diſtortion of the radii, by preſſure of the hand; ſo that the two radii of the quadrant, ſtanding at right angles to each other, exactly agreed with each other, and it continued very nearly ſo when perfectly dry. And from theſe three experiments, I concluded that an accurately-divided quadrant on a printing plate, of a foot radius, may eaſily be made to furniſh degrees and minutes in the arch of its limb,

without an error of two minutes of a degree; and that ſuch delineations may frequently be applied to good purpoſes, where the weather cannot injure the delineations, and no greater accuracy is wanted.

Experiment.

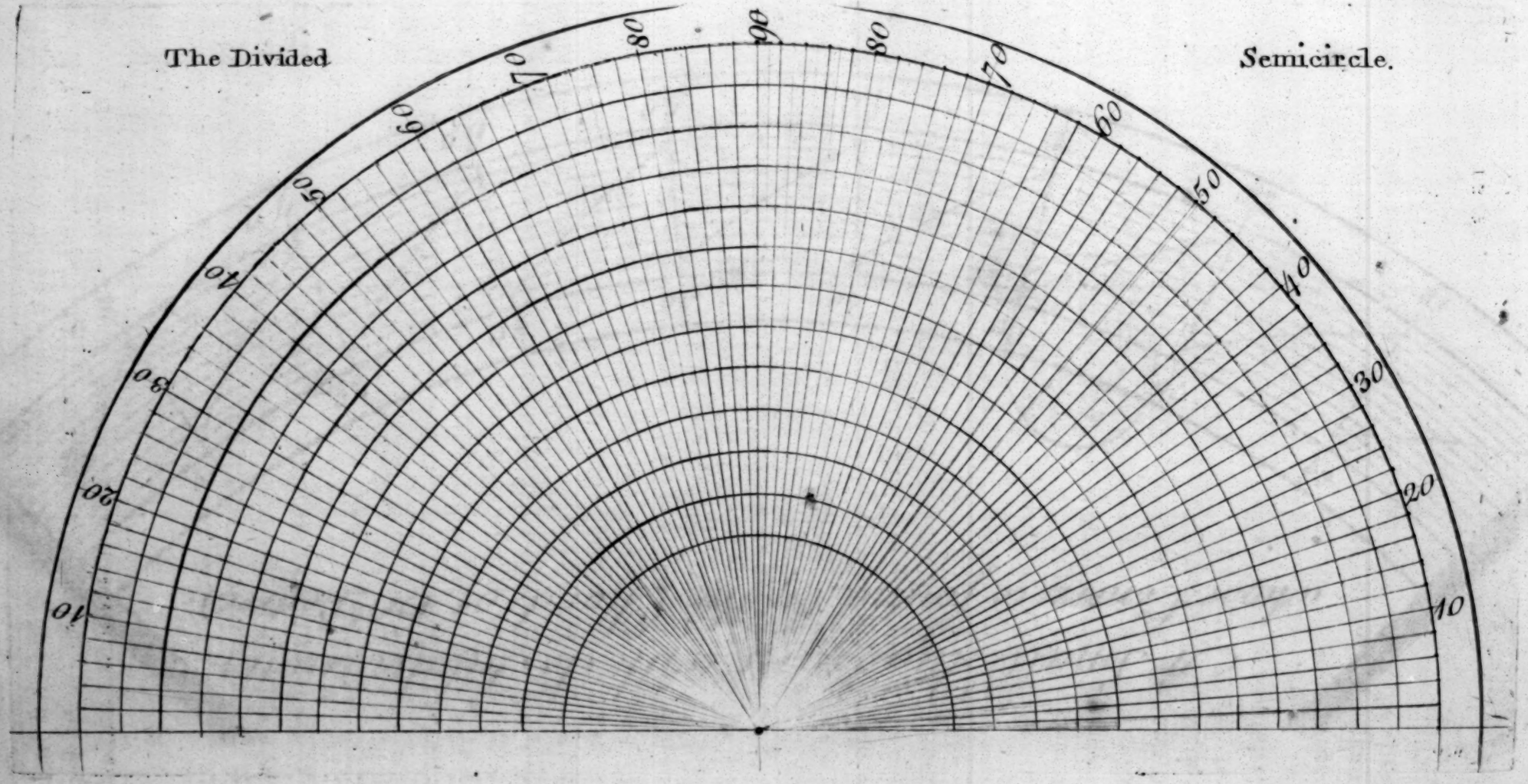
58. I found the correction for the error of the centre, as related in the ſecond experiment; and applying that correction, it determined angles, as mentioned in the third experiment.

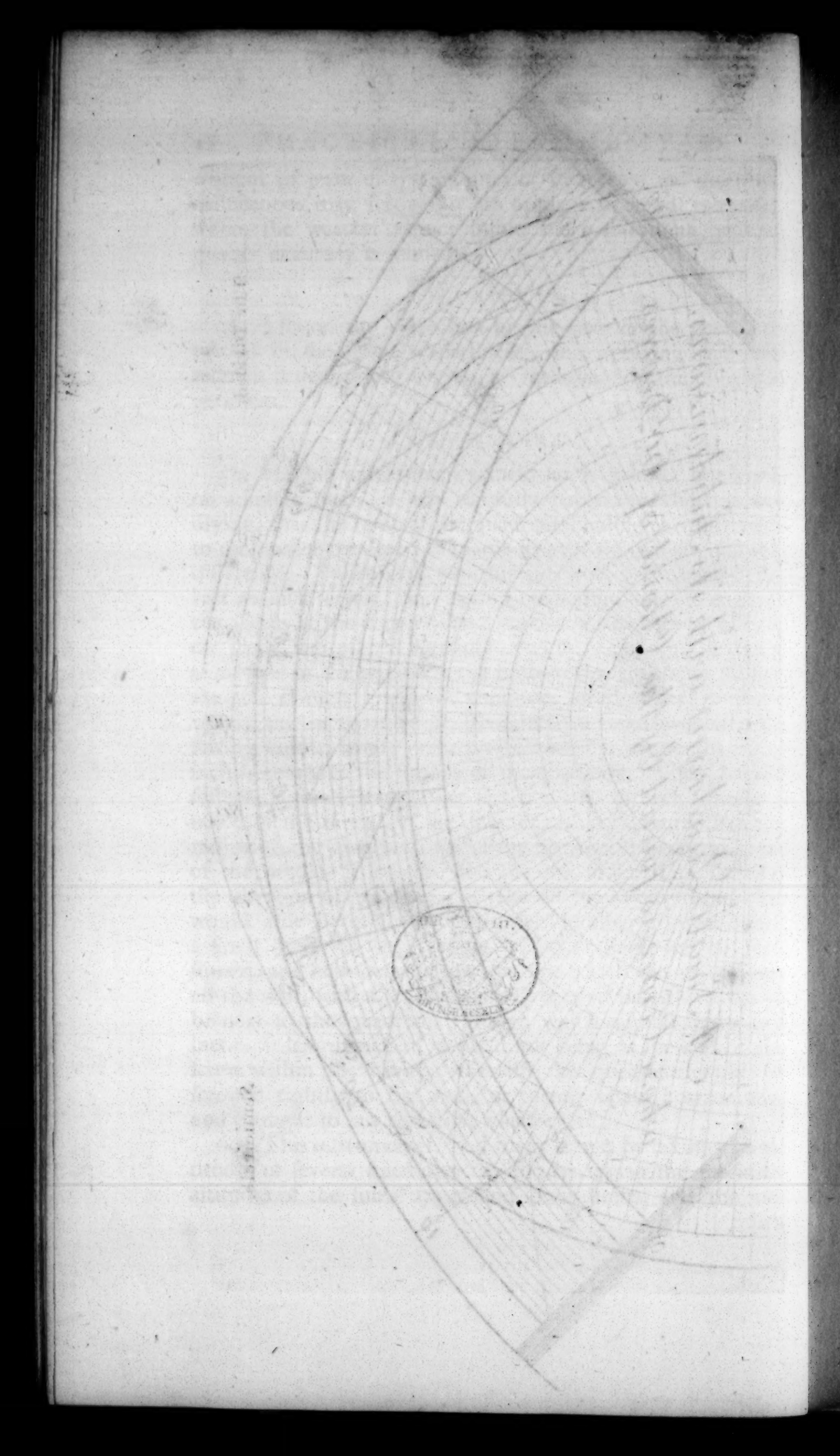
Experiment.

59. Having paſted one of theſe large printed quadrants on a ſquare board, it was carefully rubbed ſo whilſt it was drying, that the radii of the quadrant ſtanding at right angles to each other meaſured the ſame length without any ſenſible difference. I then took two thin pieces of braſs, each being half an inch ſquare; and having made four holes in each of the pieces at the corners, and another at the centre, one of the pieces was pinned carefully over the centre, and the other at the end of the radius on the limb of the quadrant. Then the hole of each braſs was continued deep enough into the wood, and in a perpendicular direction to receive a ſtraight ſewing needle, whoſe point projected about two tenths of an inch high above the ſurface of the quadrant. Then, having faſtened a ſmall lead bullet to a fine ſilk thread, I made a nooſe in the thread, at the diſtance of a little more than the radius of the quadrant, and, hanging it on the central point of the needle, it ſupplied the uſe of a plumb line, ſhewing the direction of gravity. To take off the inconvenience that would ariſe by uſing the inſtrument holding it in the hand, a ſmall piece of wood, about 5 inches long and an inch ſquare, was hollowed out on that ſide which was to be neareſt the wall, with a ſlit horizontally on that ſide which was to be next to the quadrant, through which a ſmall ſcrew two inches in length paſſed, a ſmall nut being at the end of the ſcrew within the cavity, whereby the quadrant could be ſcrewed tightly to the wall, or turned about horizontally, and brought to any elevation whatſoever.

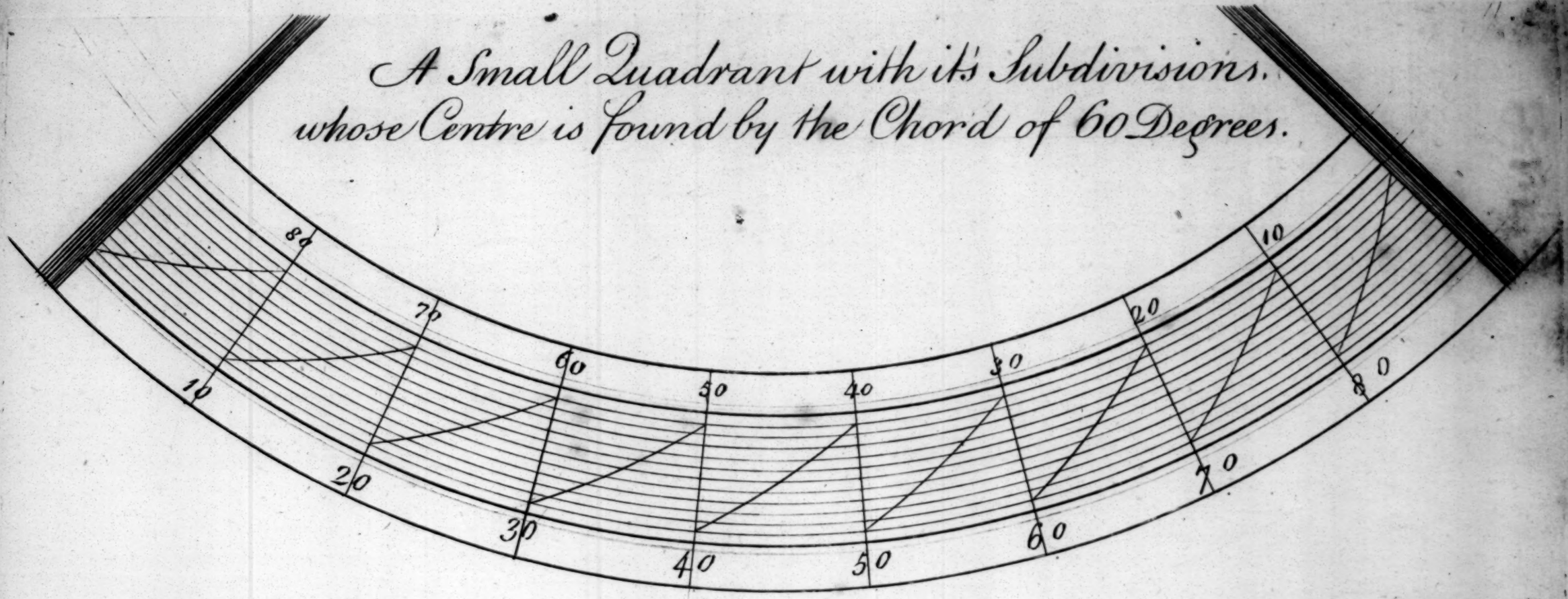
60. This inſtrument I tried many times, by taking the altitudes of ſeveral fixed objects, alſo by taking the meridian altitudes of the ſun's upper and lower limbs, and the meridian

The Divided Semicircle.





A Small Quadrant with it's Subdivisions.
whose Centre is found by the Chord of 60 Degrees.



G R
III
BRITISH MUSEUM

ridian altitudes of the fixed ſtars, and found it to anſwer without an error of two minutes of a degree, when no extraordinary care was taken in the obſervations; and by a medium of ſeveral altitudes taken near the ſame time, the altitudes might be taken by this method, without an error of one minute of a degree.

61. This temporary inſtrument may, therefore, ſupply the uſe of a more accurate one, where no other can be had, either becauſe the expence cannot be admitted, or a mathematical-inſtrument ſhop is not near; when the latitude of a place is to be taken by obſerving the ſun or ſtars, or a clock or watch is to be ſet to apparent time, or when a meridian line is to be drawn by help of correſpondent altitudes taken before and after noon, when the accuracy required doth not exceed one or two minutes of a degree in altitude. And there are many caſes, in which, when the obſervations are made at proper times, ſuch as when the celeſtial bodies are nearly eaſt or weſt, the error of one, two, or three minutes of a degree in altitude, would not render the obſervations wholly uſeleſs for geographical purpoſes.

Experiment.

62. Having drawn the ſemicircle of the ſemicircular plate to the diameter 6 inches and 86 hundredths of an inch nearly, I divided it into 90 equal parts, ſo that each diviſion was two degrees. And having drawn the radii and concentric circles as in the delineation, ſeveral prints were taken from the plate; theſe I paſted on a ſquare board of nearly the ſame dimenſion, and found it eaſy to complete the circle, with any two of theſe prints, without any conſiderable error.

Experiment.

63. Having procured two oblong pieces to be formed of clean and well-dried wood, each of half the dimenſions of this delineation, with two dove-tails of wood to make them join; after joining the parts, I paſted the two ſemicircular delineations oppoſite to each other, a print on each of the two parts of wood, and found it eaſy to make them join nearly exact, and retain a form nearly circular, without conſiderable error.

Experiment.

64. Having joined two ſuch wooden parts as beforementioned, by means of two dove-tails, I paſted a delineation of the points of the compaſs repreſenting the viſible horizon, its north and ſouth line coinciding with the joint of the parts of the wood; and when it was dry, the paper being divided at the joint, gave two ſemicircles, which, being put together, repreſented the viſible horizon, without any conſiderable error.

Experiment.

65. Having procured two oblong pieces of the dimenſions of the large ſemicircular plate mentioned in experiment 2d, I paſted them after the manner of the 6th experiment, and found the concentric circles retain nearly their true form after the paper was well dried.

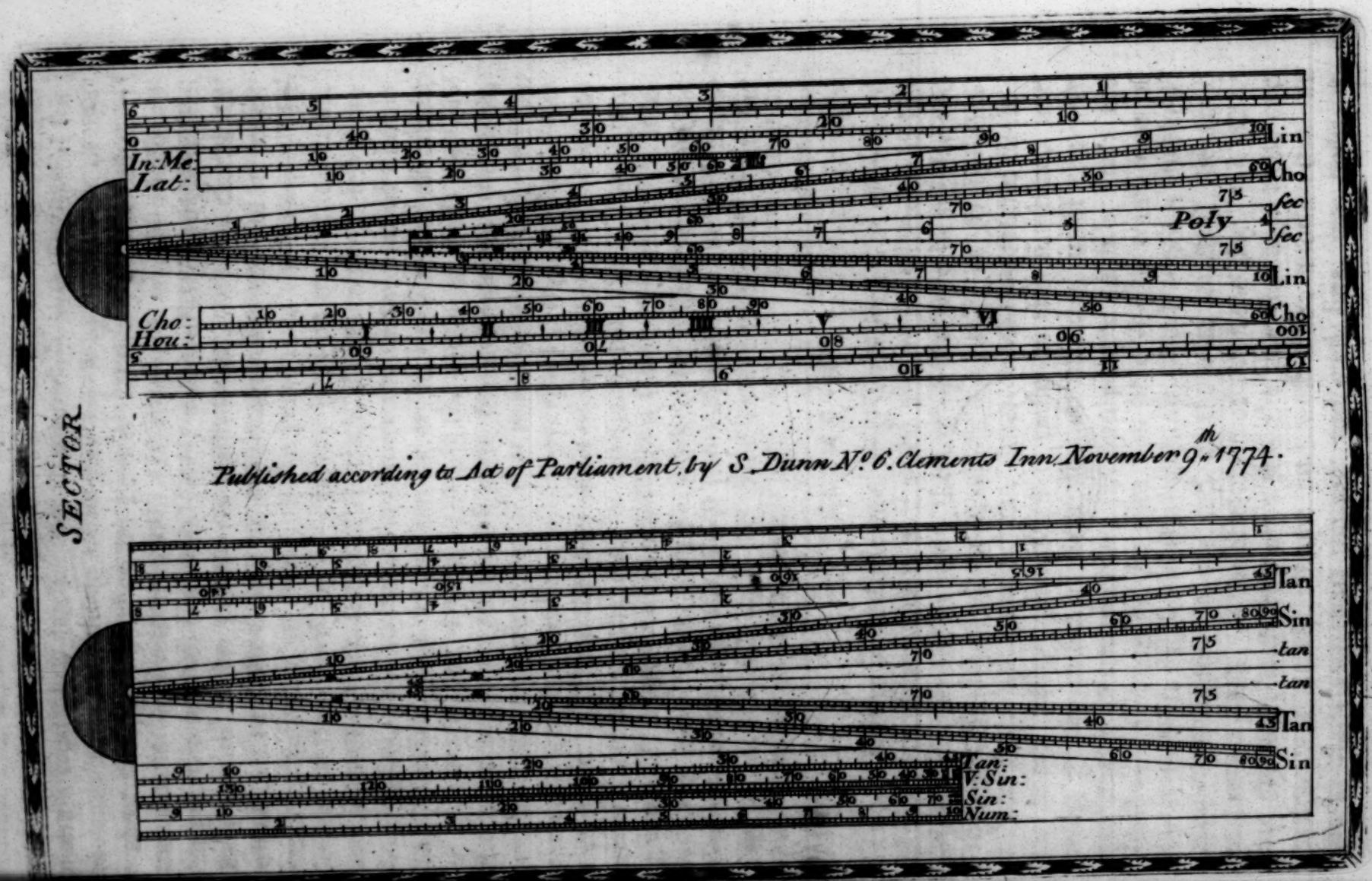
Experiment.

66. Having delineated the ſector, as in the print, and procured two ſmall pieces of fine wood ſmoothly planed, each ſix inches long, three quarters of an inch broad, and two tenths of an inch thick, I carefully paſted the two prints of the ſector on the two pieces of wood, after the manner of the former experiment; and when they were dry, the parts being ſeparated, and the two corners of the wood on which the prints were paſted being joined ſo as to touch each other, and ſupply the uſe of a joint, a ſector was formed, which anſwered, very nearly agreeing with others cut on wood and ivory, in the delineating of angles, and making of ſcales of equal parts.

Experiment.

67. Joining the two ends of this ſector to each other, ſo as to complete the line of artificial tangents, verſed ſines, ſines and numbers thereon, I worked various kinds of proportions, by help of a pair of dividing compaſſes; and the reſults came out, without any material difference from the reſults by the beſt ſectors themſelves.

68. The



Published according to Act of Parliament, by S. Dunn N.o 6. Clements Inn, November 9.th 1774.

68. The ſector is an inſtrument of ſuch general uſe in mathematical ſcience, that to treat of it properly would require a treatiſe of itſelf. In this work it may be uſeful for making and meaſuring of angles by help of a pair of dividing compaſſes, when no greater accuracy is wanted than the limits beforementioned. It may be readily made to ſupply the uſe of Gunter's lines, and working proportions by them, if it be paſted on paſte-board, or thin pieces of wood. The ſector may be advantageouſly applied in making or meaſuring an angle, as the print is in the book. The length of the chord of 60 degrees is to be made the radius, and an arch ſtruck therewith; then, if the angle is leſs than 60 degrees, it is to be taken from the ſame line of chords on the ſector, and ſet off on the arch, from which the ſides of the angle are drawn. But if the angle exceeds 60 degrees, it is proper to ſet off firſt 60 degrees on the arch, and then the overplus; or, if that is not enough, firſt one 60, then another 60, and then the overplus; and then to draw the angle. When the two ends of the ſector are joined together properly, the lines of artificial numbers, ſines, tanguents, &c. will be formed like a Gunter's ſcale. Therefore any proportions may be worked by inſpection as by the Gunter, extending for direct proportion from the firſt number to the ſecond, and the ſame extent will reach from the third number to the fourth.

Experiment.

69. Having joined the two large ſemicircles, as mentioned in the 8th experiment, I erected a ſtraight wire of a cylindrical form in the centre; the top of the wire being flat and even, and ſo adjuſted, by bending it a little, as to be equidiſtant from all parts of the outer moſt concentric circle: the board being placed on a table in the ſunſhine, and being gently raiſed or depreſſed on its ſides, until the wire was nearly perpendicular, by applying a plumb line; the ſhadow of the top of the wire was obſerved to touch one of the concentric circles in the morning, and the point marked. When the ſhadow of the top of the wire touched the ſame concentric circle, in the afternoon of the ſame day, the place was marked, and drawing a ſtraight line through theſe two points, was nearly an eaſt and weſt line; another ſtraight line at right angles to the eaſt and weſt line, was nearly a north

north and ſouth, or meridian line, over which the ſun appears when it is noon-day, ſolar time. And when any of the ſtars are over this line, they are then ſaid to be tranſiting the meridian of the place where the line is drawn.

70. This experiment is not altogether new. The uſual method of performing it is, to draw ſeveral concentric circles on the board itſelf, before the wire is erected; but here they are prepared and ready drawn, and the obſerver may uſe as many of them as he chooſes, and the medium of the obſervations will give the meridian line ſo much the more exact.

71. In the ſummer and winter months of the year, the obſervations may be made two or three hours before and after noon, without any hazard of error from the variation of the ſun's alteration of apparent declination, which ariſeth from the progreſſive motion of the earth in its orbit; but near the months of March and September, the obſervations will be moſt conveniently made, without hazarding ſuch an error, about an hour before and after noon-day.

EXPERIMENT.

72. Having placed the large ſemicircular delineations, as level as poſſible, a wire was fixed near the centre, and bent until the top of the wire was equidiſtant from each of the concentric circles, or very nearly ſo, as meaſured by a ſlip of paper, quite round. This proved that the top of the wire was nearly over the centre; and then the experiment was made after the manner of the preceding one.

EXPERIMENT.

73. Having placed a board as nearly level as poſſible, and fixed an upright wire near the middle of it; the top of the wire was made a centre by means of a thread that was not apt to ſtretch, which was carried round by the point of another wire; and thus, concentric circles being formed, the experiment was made, and the meridian line found as in the preceding experiment.

74. By this experiment the meridian of any place on land may be drawn without an error of a minute of time by two obſervations, where care and attention are applied; and where ſeveral ſuch obſervations are made, a greater degree of

of accuracy may be attained; as I have found by trials and actual practice.

EXPERIMENT.

75. The experiment was made after the ſame manner by the ſmall divided ſemicircle, and the meridian line was drawn thereby without an error of a minute of time, as verified by other proper aſtronomical obſervations.

EXPERIMENT.

76. Having procured four iron ſtaples, whoſe forms were as the figures 1, and 2, in plate of lines, two of each ſort, and two pieces of board faſtened to the cieling of my room, I drove two of theſe, of the form 1, into the boards, and nearly at right angles to, and in a direction with, the meridian. Hanging two plumb lines over theſe ſtaples, and ſliding them a little at a time, it was eaſy to bring them nearly coinciding with the plane of the meridian; which being done, it was practicable to put up two other lines, thicker and ſtronger, in the ſame places nearly; and by two other ſtaples, of the form of figure 2, driven into the floor, to ſtrain both of theſe lines tightly, in a perpendicular direction, without any conſiderable error. Theſe lines being put up are repreſented by BE and HI, or WX and TV. One of the lines might have been faſtened by a peg into the upper part of a window, as repreſented by B.

EXPERIMENT.

77. Having ſet up ſeveral of theſe lines, and withdrawing my eye to the diſtance of eight feet from one of them, and fixing it as perfectly at reſt as the body would permit, I long ſince found (for it is eleven years ſince I firſt made theſe experiments, without ever imitating others) that the contact of the ſun's limb to one of theſe threads could be obſerved without an error of two ſeconds of time, by a ſingle obſervation. And this is but an error of half a minute of a degree in the heavens.

EXPERIMENT.

78. Afterwards I ſet up two lines, as in the figures, at the diſtances from each other of 2, 4, 6, &c. to 12 feet, and withdrawing my eye, as to K or Y, found that the contact of

of the ſun's limb, and the tranſits of the ſtars, could be taken from a ſingle obſervation by theſe threads or lines, within the limit beforementioned.

Experiment.

79. Next, I procured ſmall cylinders, to be made of hard wood and of ivory, hollowed from end to end, and made to ſlide tightly on the lines; and by ſliding thoſe cylinders to a proper height, I could take the contact of the ſun's limb, by the right ſide of the line or cylinder at L, and the left ſide of the line or cylinder at M, to leſs than two ſeconds of time, and the like by the left ſide of the line or cylinder at L, and the right ſide of the line or cylinder at M; ſo that, by this artifice, I could make nine obſervations, if I choſe it, in the paſſage of the ſun, or a ſtar, acroſs theſe lines, or over the meridian. But the medium of five, or even three ſuch obſervations, would be ſufficiently exact for moſt aſtronomical purpoſes.

Experiment.

80. Having ſet up the lines at the diſtance of eight feet from each other, and meaſured the apparent path of the ſun's limb in two ſeconds of time, it amounted to a ſeventh part of the tenth of an inch nearly: and putting up the lines at the diſtance of twelve feet from each other, the motion of that limb, in two ſeconds of time, was a little leſs than the fifth part of the tenth of an inch.

Experiment.

81. Having provided myſelf with proper plumb lines, I examined how accurately the lines might be put up and ſtrained tight to the height of eight feet; and found that this could be done, without erring a fifth, or even a ſeventh, of the tenth of an inch. I might have mentioned a greater degree of accuracy, but this is ſufficient for moſt aſtronomical purpoſes. For from a great number of obſervations made by the lines, when put up with due care, they anſwered in all reſpects as accurately as they were wanted. In my firſt trials of the lines, I uſed lines of ſilk, fine, even, and ſmooth; and having put up ſeveral of them, ſome parallel to each other, others different, I found that the limb of the ſun, when it came nearly at contact with the line,

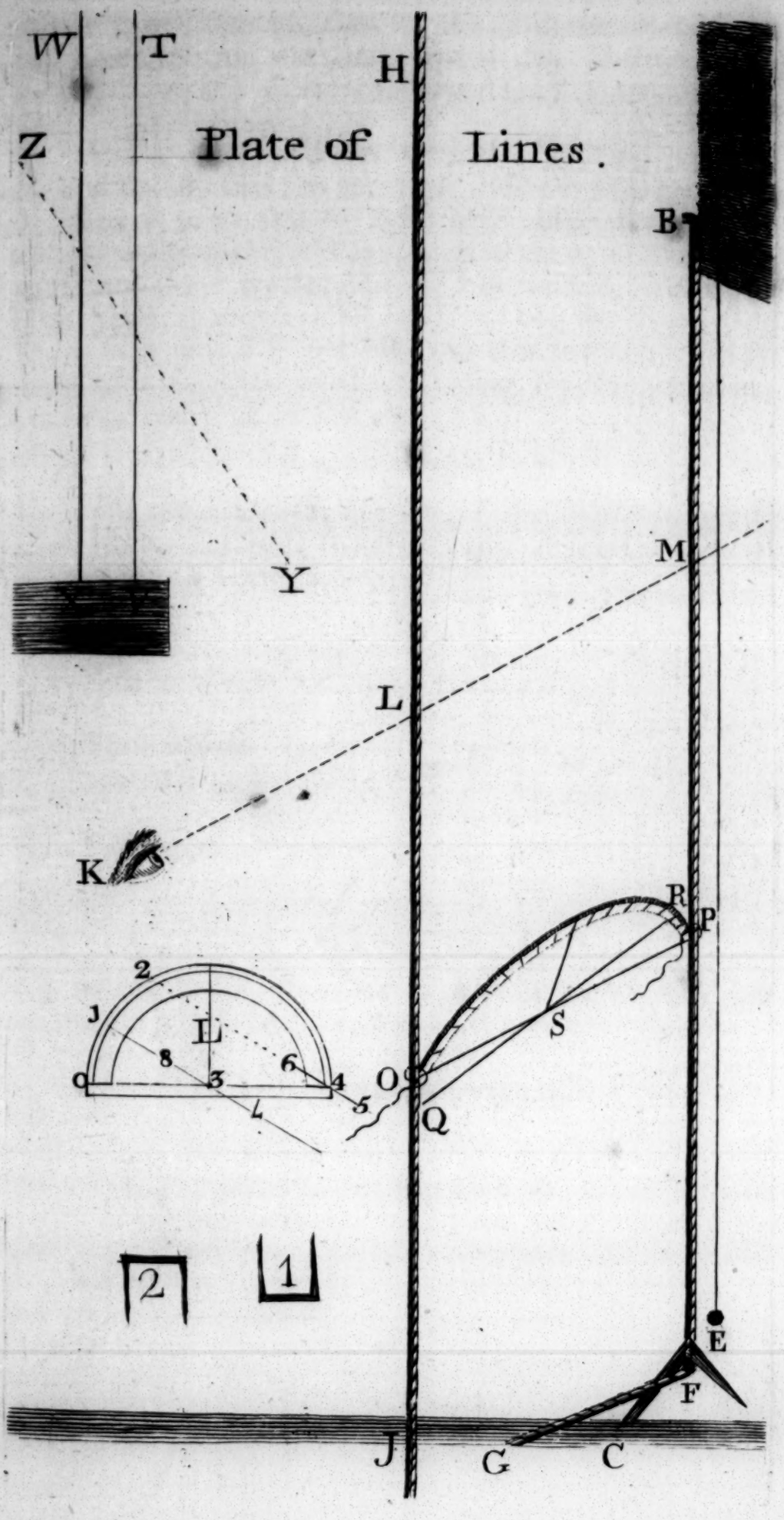

W
T
H
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Plate of Lines.
B
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line, was ſo well defined, that, by the ſteadineſs of the line, the apparent motion was ſwift enough for admitting the contact to be taken, without an uncertainty of two ſeconds of time.

82. Having diſcovered the method of putting up the lines, and finding them to anſwer at different diſtances from each other, I proceeded to make ſuch obſervations of the meridian tranſits of the celeſtial bodies by them, as thoroughly confirmed me in the opinion of their utility. For when they were put up at the diſtance of but two feet from each other, as in figures BF and HI, and that but very coarſely by help of a magnetic needle, allowing for the variation; the obſervations were as follows:

Observation.

83. 1772, March 20th, β a ſtar of the third magnitude in Canis Minor obſerved, the clock being ſet nearly to ſiderial time, and the air being unfavourable.

	h ′ ″
Immerſion, obſerved at	7 13 38
Emerſion, obſerved at	7 20 58
The medium —	7 17 18
Should have been ſouth	7 14 44
Difference — —	2 34

Observation.

84. March 24th, Procyon, a ſtar of almoſt the firſt magnitude in Canis Minor, obſerved.

Correſpondent Obſervations.

h ′ ″	h ′ ″
7 27 18	7 28 22
7 31 36	7 30 34
Medium 7 29 27	7 29 28

	h	′	″
The medium — —	7	29	27½
Should have been ſouth	7	29	20
Difference — —		2	7½

OBSERVATION.

85. The ſame evening, Regulus, a ſtar of the firſt magnitude in Leo, obſerved.

Correſpondent Obſervations.

h	′	″	h	′	″	
9	56	18	9	58	20	biſected.
9	60	17	9	58	17½	medium.
Medium 9	58	17½	9	58	19	medium.

	h	′	″
Regulus ſouth —	9	58	19
Should have been ſouth	9	56	13
Difference —		2	6

From theſe three obſervations, the clock loſt on ſiderial time at the rate of 6 ſeconds and 62 hundredths per day.

In the intermediate days, the Sun was obſerved as follows:

OBSERVATION.

86. March 22d, the Sun obſerved.

Correſpondent Obſervations.

h	′	″	h	′	″	h	′	″
12	7	41	12	8	49	12	10	13
12	15	2	12	13	43	12	12	31
Medium 12	11	21½	12	11	16	12	11	22

Biſection

	h	′	″
Biſection obſerved — —	12	11	24
One medium — —	12	11	21½
Other with clouds —	12	11	16
Another — —	12	11	22
The medium — —	12	11	21
Should have been ſouth —	12	6	50
Difference — —		4	31

OBSERVATION.

87. March 23d, the Sun obſerved.

Correſpondent Obſervations.

h	′	″	h	′	″	
12	11	12	12	12	19	
12	18	34	12	17	27	
Medium 12	14	53	12	14	53	
h	′	″	h	′	″	
12	15	5	12	16	19	
12	14	42	12	13	36	clouds.
Medium 12	14	53½	12	14	57	

	h	′	″	
One medium — —	12	14	53	
Another — —	12	14	53	
Another — —	12	14	53½	
Another — — —	12	14	57	clouds.
The medium — —	12	14	54	
Should have been ſouth —	12	6	31	
Difference — —		8	23	

OBSERVATION.

88. March 24th, the Sun obſerved.

Correſpondent Obſervations.

	h	′	″		h	′	″
	12	14	46		12	15	54
	12	22	2		12	20	47
Medium	12	18	24		12	18	21

	h	′	″	
One medium — —	12	18	24	
The other — —	12	18	21	clouds.
The medium — —	12	18	$22\frac{1}{2}$	
Should have been ſouth —	12	6	$12\frac{1}{2}$	
Difference — —		12	10	

	′	″	
The ſiderial day gains on the mean ſolar day	3	57	nearly.
The clock gained, from 22d March to 23d	3	52	

	″
The clock loſt per day by the ſun's obſ.	5
- - - - - - - - - - by the ſtars	$6\frac{2}{3}$
Difference between ſun and ſtars obſ.	$1\frac{2}{3}$

	′	″
The ſiderial day gains, &c. — —	3	57
The clock gained, from 23d to 24th March	3	47

	″
The clock loſt per day by the ſun's obſ.	10
- - - - - - - - by the ſtars	$6\frac{2}{3}$
Difference between ſun and ſtars obſ.	$3\frac{1}{3}$

89. From

89. From the near agreement of theſe obſervations, it is evident, that when the lines are put up at a greater diſtance from each other, and with greater accuracy, as they eaſily may, the tranſits may be taken ſtill more exact: although this be exact enough for moſt aſtronomical purpoſes.

90. March 25th, I ſtopt the clock whilſt it was winding up, altered its pendulum a little, and the next day, March 26th, obſerved the tranſit of Regulus in Leo, as follows:

Correſpondent Obſervations.

h	′	″	h	′	″
9	54	36	9	56	35
9	60	47	9	58	46
Medium 9	57	41½	9	57	40½

	h	′	″
The ſtar obſerved biſected	9	57	44
One medium —	9	57	41½
The other — —	9	57	40½
The medium — —	9	57	42
Should have been ſouth	9	56	13
Difference — —		1	29

Obſervation.

91. March 28th, the Sun obſerved.

Correſpondent Obſervations.

h	′	″	h	′	″
12	27	13	12	29	47
12	37	12	12	34	42
Medium 12	32	12½	12	32	14½
12	29	34	12	31	58
12	34	55	12	32	24
Medium 12	32	14½	12	32	11

One

	h	′	″
One medium — —	12	32	12½
Another — —	12	32	14½
Another — —	12	32	14½
Another — — —	12	32	11
The medium — —	12	32	13
Should have been ſouth	12	4	58
Difference — —		27	15

92. In theſe obſervations, when the ſtars were obſerved, the ſiderial time was uſed; but when the ſun was obſerved, the mean ſolar time was uſed. The method of finding both of theſe is ſhewn farther on.

93. March 30th, ſtopt the clock to give it freſh oil, took off the pendulum, put it on again, to give it ſuch a variety of accidents as may be ſuppoſed to happen to a common pendulum clock, and then ſet it going, at random, ſuppoſing it to be ſet conſiderably different from the mean ſolar time; and ſo it went on till April 4th, when the Sun was obſerved as follows:

Observation.

94. April 4th.

Correſpondent Obſervations.

	h	′	″	h	′	″	h	′	″	
	12	31	9	12	32	31	12	33	32	
	12	40	36	12	39	14	12	38	17	clouds.
Medium	12	35	52½	12	35	52½	12	35	54½	

	h	′	″
The obſerved biſection	12	35	53
One medium — —	12	35	52½
Another — — —	12	35	52½
Another — — —	12	35	54½
The medium — —	12	35	53
Should have been ſouth	12	2	50
Difference — —		33	3

Obſer-

OBSERVATION.

95. April 5th, the Sun obſerved.

Correſpondent Obſervations.

h	′	″	h	′	″	h	′	″
12	34	36	12	35	41	12	37	8
12	44	14	12	43	12	12	41	46
Medium 12	39	25	12	39	26½	12	39	27

	h	′	″
One medium — —	12	39	25
Another — — —	12	39	26½
Another — — —	12	39	27
The medium — —	12	39	26
Should have been ſouth —	12	5	30
Difference, April 5th —		33	56
Difference, April 4th —		33	3
Clock gains in a day —			53

96. Such irregularities as were found in the going of the clock, by the former obſervations, are to be expected, where there is no proviſion for the expanſion or contraction of the pendulum rod. For in the ſpring of the year, as the heat increaſes, the oil will become more fluid, and thereby accelerate the motion of the clock; whilſt, at the ſame time, the pendulum will be lengthening by the increaſing heat, and from this cauſe retard the motion of the machine. Which two cauſes, counteracting each other by irregular fits and returns, will produce the irregularities beforementioned.

97. In the making of theſe obſervations, when the ſun's tranſit was obſerved, I uſed a ſquare piece of common glaſs placed before my eye, one ſide of the glaſs being darkened by holding the flame of a candle under it, till it was dark enough to take off the greater ſtrength of the ſun's light. But as it may be more agreeable to many perſons, to take the meridian tranſit of the ſun without expoſing their ſight to the light of the ſun, I ſhall here ſhew another method how this may be done to the greateſt exactneſs, by a method

method which was diſcovered by me 12 years ſince, and after that time digeſted into a diſſertation, which was read before the Royal Society the 20th and 27th of January, the 17th of February, and 10th of March 1763, but not printed in the Tranſactions.

Experiment.

98. Having placed two round balls nearly in a line with each other and the ſun, ſo that, by the apparent motion of the ſun, the balls might come very nearly in a right line with the ſun, I received their ſhadows on a ſcreen of white paper, and obſerved the following conſequences: when the ſhadows of the balls on the ſcreen were not at contact with each other, a ſmall penumbra affected their circumferences, with ſome other circumſtances not to the preſent purpoſe. When the ſhadows came nearly at contact, the ſhadow of the ball, fartheſt from the ſun or neareſt to the ſcreen, protuberated, leaving its circular form. And finally, between the protuberating ſhadow and the other, at the point of contact, the penumbra was wholly deſtroyed, and a line of ſtrong light was formed inſtead thereof, ſo that, without any artificial means whatſoever, the inſtant when the two ſhadows became at contact with each other might be eaſily pronounced by any perſon who ſaw it, without an error of one ſecond of time. This totally takes off the penumbra of a ſhadow.

Experiment.

99. Having diſcovered this ſingular experiment relative to ſhadows, and the method of taking meridian tranſits of the ſun by help of perpendicular lines ſtrained tightly, I procured ſeveral round balls of different dimenſions and ſubſtances, each having a perforation through the centre, and fitted to move tightly on the lines. By bringing theſe balls in a line with the ſun, at or near noon, it was eaſy to take the firſt contact, or touch of the ſhadows, to a ſingle ſecond of time at moſt; and the laſt contact, or touch of the ſhadows, to the ſame exactneſs, when the ſun was not interrupted by clouds; and the middle of theſe two lines was the meridian tranſit.

Experiment.

100. I procured several cylindrical pieces of wood, ivory, and other substances, from one to two or more inches in length, and fitted to slide tightly on the lines; and these could be used together with the balls, so that the meridian transits of the sun could be taken to the greatest accuracy.

101. By this method may the meridian transits of the sun be taken from one day to another. The table of equation of time shews what hour, minute, and second, it should be by the clock or watch, when the sun is on the meridian; and the difference between the time of the sun's being on the meridian by observation, and the time it should be on the meridian by the equation of time table, shews how many minutes and seconds the clock or watch is before or after the sun; and continuing to observe from one day to another, or leaving an interval of several days, shews the gain or loss of the clock, on equal time, during any of those intervals.

102. Having invented and tried this method of putting up the lines, and making observations by them relative to the meridian transits of the sun, moon, and stars, and the variation of the magnetic needle, in 1771, I drew up a short account of it, and laid it before a certain respectable board, on which a great astronomer took this to be a common meridian line, even before the principles of it were explained.

103. But as that is not the case, and I am certain that not only this, but some other things which have had a tendency towards improvements in the art of navigation, have been proposed by me, and for which I still hope to receive some reward; I mean, a new and accurate method of dividing astronomical instruments, the introduction of proper tables for the spheroidal horizon at sea, the invention of a new quadrant by single reflection, whereby the dip of horizon can be taken most exactly, and angles to 180°, which a great instrument-maker declared impossible to be done: I shall therefore now shew what a common meridian is, and what it is not.

104. It is a meridian drawn on an horizontal plane, by means of concentric circles. This is usually performed by help of a pair of compasses, and scratching the lines on a board or paper; an upright wire placed at the centre

throws a ſhadow, from which, as in the former experiments, the meridian is drawn. But in thoſe experiments the common meridian is much improved.

105. Another ſort of meridian line is formed by placing a plate of any metal, with an aperture in it of a proper diameter, at a proper height in an upright wall, or over a window; and, having determined the poſition of the meridian line, and drawn it on the floor, the time of the ſun's tranſit over the meridian is taken by the appearance of the image or ſpot on the meridian line. This is no common meridian, becauſe it is at ſome of the moſt eminent obſervatories in Europe.

106. Plumb lines have been mentioned by ſeveral writers, but I never ſaw any mention of ſtraining them, before my own practice; and the method of ſlinging balls on them, when ſtrained, with cylinders; and the taking off of the penumbra of a ſhadow, no one elſe can have the leaſt pretenſion to, although a certain inhoſpitable writer has lately publiſhed a few trifles on looſe plumb lines, whilſt I have been preparing this work for the preſs.

107. But the method is general; for under whatever poſitions theſe lines are put up, they may be made to anſwer, if proper obſervations are made, whereby their poſitions may be firſt determined. In ſuch caſe, the error from a perpendicular direction, and the error from the true meridian, muſt be found; but there is no neceſſity for theſe things.

EXPERIMENT.

108. If the pendulum rod of a clock be of iron, it will lengthen leſs by heat than if it was of braſs, and contract leſs by cold; and therefore an iron pendulum is better than one made of braſs, becauſe the clock will go faſteſt when the pendulum is ſhorteſt, and ſloweſt when it is longeſt. But if the pendulum-rod is of white deal that is well dried, it will lengthen leſs by heat, and ſhorten leſs by cold, than if it was either braſs or iron.

EXPERIMENT.

109. The lengthening of the pendulum of a clock, by the ſummer heat, in the climate of London, will make it go

go from 16 to 18 ſeconds of time per day ſlower in ſummer than in winter. But a common watch may happen to err twenty times as much in its rate of going from one day to another, by ſuffering thoſe two extremes of ſummer's heat, and winter's cold, and by ſome other uſual accidents. And whenever either of theſe machines are applied in aſtronomical obſervations, it ſhould be exactly known how much they are before or after mean time, and how much they gain or loſe in a day; becauſe otherwiſe (however truly they are ſet before an obſervation is made) at the making of the obſervation, the clock or watch, which is to indicate the time, may give it erroneouſly, and thereby render the obſervation uſeleſs.

110. The table of the equation of time is to be met with in ſeveral good publications, particularly the Nautical Ephemeris, and the Connoiſſance des Temps; and perſons who have not thoſe elaborate performances may have the equation near enough for moſt purpoſes, at the end of this work.

111. The apparent gaining of the fixed ſtars, whereby they tranſit the meridian of any place of obſervation, ſooner than the expiration of mean ſolar time, for any number of days, not exceeding a month, is nearly as in the following table.

Days	′	″	Days	h	′	″
0	0	0	16	1	2	54
1	3	56	17	1	6	50
2	7	52	18	1	10	46
3	11	48	19	1	14	42
4	15	44	20	1	18	38
5	19	39 ½	21	1	22	34
6	23	35	22	1	26	30
7	27	31	23	1	30	26
8	31	27	24	1	34	22
9	35	23	25	1	38	17 ½
10	39	19	26	1	42	13 ½
11	43	15	27	1	46	9
12	47	11	28	1	50	5
13	51	7	29	1	54	1
14	55	3	30	1	57	57
15	58	58 ½	31	2	1	53

112. There-

112. Therefore, if you take the exact time of the meridian tranſit of any fixed ſtar, on any day, and note that time by the clock; and, having let the clock go on for one, two, or more days, until you can take the meridian tranſit of the ſame ſtar again, and note its time by the clock: if the ſtar has gained nearly agreeing with the correſpondent numbers in this table, it goes nearly to mean ſolar time, and is fit to be applied, and farther adjuſted in the making of an aſtronomical obſervation.

113. The magnetic needle is made of different lengths and of different forms; but, whatever they are, it ſhould be of ſteel, perfectly ſtraight, ſo poiſed as to keep an horizontal poſition, and its cap, whether of agate or braſs, ſhould be well formed, and permit the needle to traverſe or play eaſily and freely.

Experiment.

114. Having procured ſeveral of thoſe needles, of different forms and lengths, from three to eighteen inches in length, I fixed an upright wire, which was exceedingly well pointed, into a board of proper thickneſs, and, faſtening the end of a fine thread by a nooſe to the wire, ſtretched the thread, and made it coincident with the direction of the needle, when it became at reſt. I tried all theſe different needles, and could find no difference in their directions.

Experiment.

115. Having put ſeveral ſmall beads on a thread, ſo that, moved tightly thereon, by means of the hooks before deſcribed, the thread was brought as nearly as poſſible to a perpendicular direction, and there ſtrained tightly. The centre of one of the large ſemi-circles was brought as near as poſſible to the thread, and the beads ſo moved as to cauſe them to project their ſhadows on the concentric circles, in the forenoon, after the manner of experiment 19th, and the like in the afternoon; from which, the eaſt and weſt, and meridian line, was found.

116. From theſe two latter experiments, although they are made without aſtronomical inſtruments, I conclude that a meridian line may be drawn, when care and good judgement are applied, without an error of ten minutes of a degree on the horizontal plane, at the moſt, which is ſufficiently exact

exact for making a great number of very uſeful magnetical experiments relative to the variation of the needle, and the change of that variation.

EXPERIMENT.

117. Having drawn a meridian line after the manner before deſcribed, and confirmed it by a more correct aſtronomical obſervation, I applied ſeveral of theſe needles to the meridian, and found they deviated from the true meridian at London, for this year alike nearly.

EXPERIMENT.

118. I took the variation of the needle, at London, 100 miles north eaſtwardly of London, and ſouthweſtwardly of London, for the ſame year; and found the variation exactly agreeing with thoſe lines of equal variation produced according to their obſerved poſitions and directions eaſtward, from the Atlantic ocean, and the Britiſh channel.

EXPERIMENT.

119. Having cut off the ſharp end of a wire pin, and placed it upright at the centre between the two paſted ſemicircles, and brought the joint to the meridian, by means of a thread, as before deſcribed, the variation was eaſily taken. If the pin was placed at the beginning of the ſemicircle, as at 4 in plate of lines, the direction of the meridian being O, 4, the variation of the needle would be O, 2, in which arch, every two degrees would now be but one degree, becauſe the angle at the circumference 2, 4, O, would be equal to the angle at the centre 1, 3, O, the lines 2, 4, and 1, 3, being parallel. After the ſame manner, if ORP was the ſemicircle, OP the meridian, and QR the needle, PR would be the variation.

EXPERIMENT.

120. Having provided a proper ſituation where I could ſee thoſe two places of the horizon, where the ſun apparently immerged to, and emerged from, at ſun riſing and ſetting: I placed the large ſemicircle beforementioned in a proper poſition horizontal, and took the ſun's bearing at each of theſe times; and drawing the line between theſe two bearings, the meridian

meridian line was found, and the magnetic needle applied, which gave the variation.

121. This experiment ſhould be made when the ſun is towards its greateſt north or ſouth limits, or in the more ſummer or winter quarters of the year.

122. The meridian line being continued quite round the earth's ſurface, forms a circle nearly, but, more properly ſpeaking, an ellipſis flattened towards the earth's pole; and the plane of this meridian being extended into the heavens, points out the circumference of a circular meridian in the heavens. An indefinite number of ſuch terreſtrial meridians circumſcribe the earth, and the heavens; and as the celeſtial bodies appear on one of theſe celeſtial meridians by the earth's rotation, they are ſaid to tranſit over the terreſtrial meridian that is under it.

123. The equinoctial line is a great circle of the earth, 90 degrees diſtant from either of the earth's poles. The number of degrees and minutes, which a place of the earth or the ſea is from the equinoctial, reckoning on the meridian of that place, ſo many degrees and minutes of latitude is that place ſaid to be in.

124. The equinoctial line, being extended into the heavens, points out the circumference of the equator; and as many degrees and minutes as any celeſtial body appears diſtant from the equator, ſo many degrees and minutes of declination it is ſaid to have, whether it be northward or ſouthward of the equator.

125. The longitudes of places on the earth and ſea are counted wholly by help of the meridians; for as the meridians of all places are ſuppoſedly continued to the equinoctial, ſo the number of degrees and minutes which the meridians of places happen to be from each other, when they are prolonged to the equinoctial; as many degrees and minutes difference of longitude, or longitude itſelf, are thoſe places commonly ſaid to be from eath other, whether the places be eaſt longitude or weſt longitude from each other.

126. Whilſt the progreſſive motion of the earth in its orbit, and its diurnal rotation, is producing an appearance of the change of the ſun's declination from day to day, it likewiſe produceth an appearance of the ſun's progreſſive motion

tion through the heavens, ſo as thereby to make one whole revolution amongſt the fixed ſtars in a year.

127. This great circle is called the ecliptic in the heavens, and is the apparent yearly path of the ſun amongſt the fixed ſtars. The greateſt deviation of the ſun from the equator, both north and ſouth, is called the obliquity of the ecliptic. This is now 23° 28′ nearly.

128. The celeſtial bodies are higheſt elevated above the horizon, at the inſtant of ſolar noon; this is for the ſun, and for all the others, when they are over the meridian, or north and ſouth line of the place of obſervation, except ſuch as paſs through the heavens, ſo as to change their declination very faſt. Theſe meridian elevations are called meridian altitudes, and they are obſerved in finding the latitudes of places.

129. There are four caſes of finding the latitude by the meridian altitudes of the celeſtial bodies:

Namely, 1. North latitude and north declination.
2. North latitude and ſouth declination.
3. South latitude and ſouth declination.
4. South latitude and north declination.

130. The complement of an arch that is leſs than 90 degrees, is its remainder to 90 degrees. But the complement of an arch that is more than 90 degrees, is its remainder to 180 degrees.

131. The zenith of a place is that point of the heavens directly over a perſon at that place. The nadir of a place, is that point of the heavens directly under a perſon's feet at that place. When the meridian altitude of any celeſtial body is ſubtracted from 90°, the remainder is uſually called the zenith diſtance. But, ſtrictly ſpeaking, when any other altitude of a celeſtial body is ſubtracted from 90°, the remainder is the zenith diſtance.

132. When a perſon looks towards the north at noon, and he is in north latitude, and the ſun has north declination, theſe three are ſaid to be of one name; otherwiſe not. Hence the poſition of the ſun at noon, the latitude and declination are either ſaid to be alike, or unlike, as the caſe happens to be.

133. The beſt way of determining whether the declination is to be added or ſubtracted, when the zenith diſtance

is

is taken, is to make a ſmall diagram or figure; but without making ſuch, it may found by the following verbal rules.

134. When the declination and zenith diſtance are both of the ſame name, their difference is the latitude of the place, and it is of the ſame name with the declination. But, if the zenith diſtance and declination are not of the ſame name, their ſum will be the latitude of the place, and it is of the ſame name with the declination. But it will be more plain in moſt caſes to find the meridian altitude of the equator, which is the complement of the latitude to 90°: becauſe by this method, whether the upper or lower limb of the ſun be obſerved, the calculation will be leſs confuſed; and the allowance for refraction, parallax, ſemidiameter, &c. will be more natural.

135. In obſerving the altitudes of the celeſtial bodies on land, whether the inſtrument be regulated by a plumb line or ſpirit level, whatever the altitude is as ſhewn by the inſtrument, that is ſet down for the obſerved altitude. This altitude muſt be corrected for the effects of refraction and parallax; and when the lower limb of the celeſtial body is obſerved, its ſemidiameter muſt be added: but when the upper limb is obſerved, the ſemidiameter muſt be ſubtracted; and this gives the true altitude of the centre of the celeſtial body above the horizon of the place of obſervation.

136. At ſea, the ſurface of the water, near the ſhip in which the obſerver is, may be conſidered as being nearly in the horizon of that place. And as the obſerver is more elevated by being on the ſhip, he thereby can look round him, and ſee places in the heavens a little under the apparent or viſible horizon of the place where he is; and this is uſually called the dip of horizon, and allowed for according to the height of the eye of the obſerver, above the ſurface of the ſea, by means of a ſmall table for that purpoſe.

137. The horizontal refraction in the latitude of London is conſiderable; it amounts to 31 minutes of a degree in ſummer, and 35 minutes in winter; and therefore as the ſun's diameter ſubtends 32 minutes at a medium, and the ſun is apparently elevated thus much at the horizon higher than it really is, it follows, that when we ſee the ſun juſt above the horizon, it is then in reality but beginning to

come

come above the horizon; and thus much are we deceived in our viſion by the interpoſition of horizontal vapours.

138. It is an uſual practice at ſea, to reject the effects of refraction. This will occaſion ſome error, when the altitudes are but ſmall. On land, aſtronomers never reject the refractions; but as the effect of the ſun's horizontal parallax is now generally concluded to be but ſmall, and that it cannot, even near the horizon, produce a greater effect than 8 ſeconds and half, of a degree, and at different altitudes from the horizon to the zenith is much leſs and inconſiderable; it is therefore not unuſual, in the more coarſe and rude obſervations, to neglect the effects of parallax in obſervations of the ſun. And as to the fixed ſtars, they have no parallax that can be obſerved.

139. Having given the principles, we now proceed to the calculation of the latitudes of places, from obſervations ſuppoſed to have been made both on land and at ſea. On land, the latitude may be taken either by meridian altitudes of the ſun, or by meridian altitudes of the pole ſtar. The altitude of the ſun, or pole ſtar, may be taken on land, either by a braſs quadrant fixed to a pillar, but moving eaſily to any poſition, elevation, or depreſſion; or, for want of ſuch an inſtrument, it may be taken with tolerable accuracy by the delineated quadrant belonging to this work, before deſcribed, when it is properly paſted and mounted. But at ſea, the latitude is beſt found by taking meridian altitudes of the ſun with Hadley's quadrant; in the doing of which, it is the uſual practice, to bring the ſun's lower limb to the horizon of the ſea, and add the ſemidiameter of the ſun, which mariners uſually reckon at a medium 16 minutes of a degree; but it is frequently a little more or leſs, according to the tables. Aſtronomers on land uſually obſerve the upper limb of the ſun, this appears loweſt when viewed through a teleſcope that inverts the object; in ſuch caſe, the ſemidiameter of the ſun is ſubtracted from the obſerved altitude.

140. Case 1. May 20th, 1775, on land, the meridian altitude of the Sun's upper limb was observed 61° 17′.

Required the latitude of the place of observation?

CALCULATION.

	°	′	″
Altitude of the upper limb observed — —	61	17	0
Refraction to be subtracted — — —	0	0	30
Upper limb cleared from refraction — —	61	16	30
Parallax inconsiderable. Semidiameter subtract	0	15	51
True altitude of the Sun's centre — — —	61	0	39
Sun's declination north — — — subtract	20	0	0
Meridian altitude of the equator — —	41	0	39
Its complement, the latitude of the place north	48	59	21

141. Case 2. January 20th, 1775, on land, the meridian altitude of the Sun's upper limb was observed 15° 25′.

Required the latitude of the place of observation?

CALCULATION.

	°	′	″
Altitude of the upper limb observed — —	15	25	0
Refraction to be subtracted — — — —	0	3	16
Upper limb cleared from refraction — —	15	21	44
Parallax inconsiderable. Semidiameter subtract	0	16	18
True altitude of the Sun's centre — —	15	5	26
Sun's declination south — — — add	20	5	40
Meridian altitude of the equator — —	35	11	6
Its complement, the latitude of the place north	54	48	54

142. Case

142. Caſe 3. November 15, 1775, on land, the meridian altitude of the Sun's upper limb was obſerved 74° 15′.

Required the latitude of the place of obſervation?

CALCULATION.

		°	′	″
Altitude of the upper limb obſerved — —		74	15	0
Refraction to be ſubtracted — — —		0	0	15
Upper limb cleared from refraction — —		74	14	45
Parallax inconſiderable. Semidiameter ſubtract		0	16	14
True altitude of the ſun's centre — —		73	58	31
Sun's declination ſouth — — ſubtract		18	32	27
Meridian altitude of the equator — —		55	26	4
Its complement, the latitude of the place ſouth		34	33	56

143. Caſe 4. July 15, 1775, on land, the meridian altitude of the Sun's upper limb was obſerved 15° 26′.

Required the latitude of the place of obſervation?

CALCULATION.

		°	′	″
Altitude of the upper limb obſerved — —		15	26	0
Refraction to be ſubtracted — — —		0	3	12
Upper limb cleared from refraction — —		15	22	48
Parallax inconſiderable. Semidiameter ſubtract		0	15	48
True altitude of the ſun's centre — —		15	7	0
Sun's declination north — — — add		21	33	20
Meridian altitude of the equator — —		36	40	20
Its complement, the latitude of the place ſouth		53	19	40

But when the Hadley's quadrant is uſed, there is another way of making the obſervation for the latitude, either on land or at ſea, which is as follows. A ſmall quantity of clear water is put into any open veſſel, and a ſmall quantity of wheat-flour is put into it; and ſtirring them together, the flour thickens the water, and thereby makes it leſs affected by the motion of the air or wind. Then, inſtead of bringing the image of the ſun's lower limb down to the edge of a viſible horizon, as is the practice of mariners at ſea, the image, by reflection from the glaſſes of the quadrant, is brought down, to cover the image reflected from the water: and half of the angle which is ſo taken is the altitude of the ſun's centre; which, being cleared from refraction, and the declination applied thereto as the caſe requires, gives the latitude. The making of theſe obſervations more accurately is treated of farther on, it being here only intended to ſhew the method of taking the latitude to ſome tolerable accuracy.

The ſame examples being calculated by the zenith diſtances, will be as follows:

144. Caſe 1ſt.

	°	′	″
Altitude of the upper limb obſerved — —	61	17	0
Co altitude of the upper limb obſerved —	28	43	0
Refraction to be added — — —	0	0	30
Zenith diſtance cleared from refraction —	28	43	30
Parallax inconſiderable. Semidiameter add	0	15	51
True zenith diſtance of Sun's centre, ſouth,	28	59	21
Sun's declination north — — — add	20	0	0
Latitude of the place north — —	48	59	21

145. Caſe 2d.

	°	′	″
Altitude of the upper limb obſerved — —	15	25	0
Co altitude of the upper limb obſerved —	74	35	0
Refraction to be added — — —	0	3	16

Zenith

		°	′	″
Zenith diſtance cleared from refraction —		74	38	16
Parallax inconſiderable. Semidiameter	add	0	16	18
True zenith diſtance of ſun's centre, ſouth,		74	54	34
Sun's declination ſouth — —	ſubtract	20	5	40
Latitude of the place north — —		54	48	54

Having taken many meridian altitudes of the ſun, by obſerving the upper limb, the lower limb, and the centre, and calculated the latitude therefrom, the altitudes having been taken by one of the large printed quadrants before deſcribed, paſted on board, and properly mounted, it has determined the latitude of London within two minutes of a degree frequently; and when ſeveral meridian altitudes have been obſerved the ſame day, and the medium of them taken, the latitude has been determined thereby to leſs than a minute of a degree. The like has been done by meridian altitudes of ſeveral of the fixed ſtars. And therefore ſuch an inſtrument may be uſeful on land for many geographical purpoſes, when and where no better can be had to be uſed.

146. Caſe 3d.

		°	′	″
Altitude of the upper limb obſerved —		74	15	0
Co. altitude of the upper limb obſerved —		15	45	0
Refraction to be added — — —		0	0	15
Zenith diſtnace cleared from refraction —		15	45	15
Parallax inconſiderable. Semidiameter	add	0	16	14
True zenith diſtance of Sun's centre, north		16	1	29
Sun's declination ſouth — —	add	18	32	27
Latitude of the place ſouth — —		34	33	56

147. Caſe

147. Caſe 4th.

	°	′	″
Altitude of the upper limb obſerved —	15	26	0
Co. altitude of the upper limb obſerved —	74	34	0
Refraction to be added — — —	0	3	12
Zenith diſtance cleared from refraction —	74	37	12
Parallax inconſiderable. Semidiameter add	0	15	48
True zenith diſtance of Sun's centre, north	74	53	0
Sun's declination north — — ſubtract	21	33	20
Latitude of the place ſouth — —	53	19	40

148. As the obliquity of the ecliptic is a kind of foundation on which the ſun's declination ſtands, and it is now generally thought to be in a ſtate of diminution; we proceed to examine ſome of the moſt antient and correct obſervations that have been made, to determine whether theſe concluſions of aſtronomers are true or not.

149. The moſt antient aſtronomer, who is recorded to have made any obſervations for determining the ſun's greateſt north and ſouth declination, to any conſiderable degree of accuracy, was Pytheas at Marſeilles. And from his obſervations, the obliquity of the ecliptic is ſaid to have been found 23° 52′ 41″, 324 years before Chriſt.

Observation.

150. The learned profeſſor Wolfius, in his Elementa Matheſeos Univerſæ, tom. III, ſpeaking of this aſtronomer, cites the following paſſage concerning him, from Vita Peireſcii. "Pytheas Maſciliæ umbram ſolſtitialem ad gnomonem obſervavit ut $213\frac{1}{5}$ ad 600 ſeu ut $31951\frac{1}{2}$ ad 90000; Gaſſendus cum Peireſcio ibidem, A. 1636, ut 31950 ad 90000."

151. From this expreſſion, the gnomon, which Pytheas obſerved by, was 90000 equal parts; it caſt a ſhadow; the length of the ſhadow was $31951\frac{1}{2}$ of theſe parts; and the ſun's greateſt declination, at that time, may be determined as follows:

As

As 31951,5 : 90000, :: radius : tangent 70° 27′ 15″. From which ſubtract the ſun's ſemidiameter 15′ 47″, and the refraction 20″, and adding the parallax in altitude 3″, gives the true altitude of the ſun's centre 70° 11′ 11″. The latitude of Marſeilles is 43° 17′ 45″, and co. latitude 46° 42′ 15″, which taken from 70° 11′ 11″ gives 23° 28′ 56″, the obliquity of the ecliptic, by one of Pytheas's obſervations.

152. If we take the other of them, it will be as 213,2 : 600, :: radius : tangent 70° 26′ 17″, from which taking the ſemidiameter and refraction, and adding the parallax as before, gives 70° 10′ 13″ the true altitude of the ſun's centre, from which ſubtracting the co. latitude 46° 42′ 15″ gives 23° 27′ 58″, the obliquity of the ecliptic by the other of Pytheas's obſervations. And from the medium of both, the obliquity of the ecliptic was 23° 28′ 27″, 324 years before Chriſt, or 2098 years ago. This is a remarkable circumſtance, as the obſervation of Gaſſendus confirms the preceding ones of Pytheas.

Observation.

153. Doctor Halley, in Philoſophical Tranſactions abridged, N° 215, ſays, that when Gaſſendus obſerved the ſolſtitial ſhadow at Marſeilles, the proportion of the ſhorteſt ſhadow to the gnomon, was as 31751 to 89428. Therefore, as 31751 : 89428 :: radius : tangent 70° 27′ 10″, which being but 5″ leſs than the former of Pytheas's, muſt give the obliquity of the ecliptic 23° 28′ 51″.

Observation.

154. In the year 1715, M. de Loville went to Marſeilles, and having made obſervations for determining the obliquity of the ecliptic, found it 23° 28′ 24″. This being compared with the obſervation of Gaſſendus amounts to but a diminution of 27″ in 79 years, which is 20‴ and half in three years. But it ſhould be obſerved that as, in antient times, aſtronomers were unacquainted with the quantities of refraction and parallax at different altitudes, which lead them into various erroneous concluſions; ſo they are not now exactly agreed concerning refractions.

Obſer-

Observation.

155. In 1487, Bernard Walther, at Nuremberg, measured the chord of the ſun's diſtance from the zenith, on the longeſt day, by the parallactic inſtrument of Ptolomy, and found it 44890 parts of the radius 100000. The ſame was confirmed by ſeveral years ſucceeding obſervations. And the ſhorteſt day of the ſame year, the chord of the ſun's zenith diſtance was 118790, and this was confirmed by other obſervations, as recorded Phil. Tranſ. N° 190. Half of the former chord is 22445, and of the latter is 59395; from which, the ſine of half the former zenith diſtance was 2244500, the half zenith diſtance 12° 58′ 14″, the zenith diſtance 25° 56′ 28″, and the altitude 64° 3′ 32″, the ſine of half the latter zenith diſtance was 5939500, the half zenith diſtance 36° 26′ 16″, the zenith diſtance 72° 52′ 32″, and the altitude 17° 7′ 28″. The effects of refraction and parallax, to be ſubtracted from the former altitude, is 29″; and from the latter is 2′ 54″. So the true ſummer altitude was 64° 3′ 3″; and the true winter altitude was 17° 4′ 34″; the difference is 46° 58′ 29″; and the half difference 23° 29′ 14″, the obliquity of the ecliptic by theſe obſervations.

Observation.

156. In 1586 and 1594, Tycho Brahe, at Uraniburg, obſerved the ſun's greateſt meridian altitude 57° 35′ 36″, and the leaſt meridian altitude 10° 41′ 10″; the effects of refraction and parallax to the former altitude is 30″, and to the latter 4′ 32″, whence the true altitudes were 57° 35′ 6″, and 10° 36′ 38″, the difference is 46° 58′ 28″, and its half is 23° 29′ 14″, the obliquity of the ecliptic.

Again, the latitude of Uraniburg is 55° 54′ 15″, and co. latitude 34° 5′ 45″, which taken from 57° 35′ 6″, gives 23° 29′ 21″, the obliquity of the ecliptic, independent of the winter meridian altitude.

Observation.

157. In the years 1594, 1595, 1596, and 1597, Edward Wright, inventor of the ſea chart, falſely attributed to Mercator, made careful obſervations of the meridian altitudes of the pole ſtar above and below the pole, and of the ſun's greateſt

greateſt meridian altitude, at London, as recorded in his Treatiſe of Navigation; and ſays, the pole ſtar's meridian altitude above the pole was 54° 24′ 30″, and below the pole 48° 39′ 30″; the refraction of the former altitude being 39″, and of the latter 45″, gives the true altitude above the pole 54° 23′ 51″, and below the pole 48° 38′ 45″, the medium of which is 51° 31′ 18″, the latitude of the place in London where he obſerved, which is not half a mile northward of St. Paul's. By theſe obſervations, the ſun's greateſt meridian altitude obſerved was 61° 58′ 0″, from which ſubtracting the refraction 29″, and adding the parallax 4″, gives 61° 57′ 35″ the true altitude of the ſun's centre, which leſſened by the co. latitude 38° 28′ 42″, gives 23° 28′ 53″ for the obliquity of the ecliptic in 1597, by Edward Wright's obſervations. He expreſsly ſays, "I "found the greateſt height thereof at that time here in "London, &c." by which no doubt he meant within the city; whoſe wall north of St. Paul's not exceeding 17″ of a degree, if it was made there, his true latitude was 51° 30′ 57″, and the obliquity of the ecliptic was then 23° 28′ 32; but if it was made near St. Paul's, the obliquity of the ecliptic was 23° 28′ 15″.

Observation.

158. In 1646, Ricciolus, at Bologna in Italy, obſerved the ſun's greateſt meridian altitude 69° 0′ 10″, from which take the refraction 22″, gives 68° 59′ 48″ the true meridian altitude. The latitude is 44° 29′ 36″, and the co. latitude 45° 30′ 24″, which taken from 68° 59′ 48″, gives 23° 29′ 24″ for the obliquity of the ecliptic by this obſervation.

Observation.

159. In 1655, M. Caſſini, at Bologna, obſerved the obliquity of the ecliptic 23° 29′ 15″; but the refraction he uſed is not named. Both of theſe obſervations, made at Bologna, give the obliquity of the ecliptic more than it was ſixty or ſeventy years before, by Tycho Brahe's obſervations at Uraniburg; and as much as it was 160 years before at Nuremberg, by the obſervations of Bernard Walther.

Observation.

160. In the years 1658, 1659, 1660, 1661, and 1662, Gabriel Mouton, at Lyons, obſerved the pole ſtar's greateſt height above the pole, 48° 17′ 11″; and below the pole, 43° 15′ 30″; the refraction to the former being 48″, and to the latter 58″; ſo the true altitudes were 48° 16′ 23″ and 43° 14′ 33, the medium of which is 45° 45′ 28″, which is but 23″ leſs than it has been lately found. From theſe obſervations, the medium of the ſun's greateſt meridian altitudes obſerved by a large quadrant for four ſucceeding years was 67° 44′ 30″, and by a large gnomon 67° 43′ 26″, each of which being leſſened by the refraction 23″, gives the altitude 67° 44′ 7″ and 67° 43′ 3″, from which ſubtracting his co. latitude 44° 14′ 28″, gives 23° 29′ 35″ for the obliquity of the ecliptic by the quadrant, and 23° 28′ 33″ by the gnomon, the medium being 23° 29′ 4″.

Observation.

161. In 1672, M. Richer, in the Iſle of Cayenne, obſerved the obliquity of the ecliptic 23° 28′ 48″. This obſervation is remarkable for its having been made within 5 degrees of the equinoctial line, where, on account of the great altitudes of the ſun, it may be ſuppoſed to have been the more out of any danger from refraction.

Observation.

162. In 1681, the greateſt meridian altitude of the ſun's upper limb was obſerved, as recorded by De la Hire, at Paris, 64° 55′ 24′; from which ſubtract the ſemidiameter 15′ 47″, and refraction 26″, adding the parallax 4″, gives the ſun's true altitude 64° 39′ 15″. At the ſame place, the leaſt meridian altitude of the ſun's upper limb was 18° 0′ 24″; from which ſubtract the ſemidiameter 16′ 19″, and refraction 2′ 41″, and add the parallax 8″, gives the ſun's true altitude 17° 41′ 32″, which taken from 64° 39′ 15″, gives 46° 57′ 43″, whoſe half is 23° 28′ 51″; the obliquity of the ecliptic deduced from theſe obſervations.

OBSERVATION.

163. In 1686, M. Wertzelbaur, at Nuremberg, obſerved the ſun's greateſt meridian altitude 64° 2′ 25″; and at the ſame place, and the ſame year, obſerved the leaſt meridian altitude 17° 7′ 10″. The effect of refraction and parallax for the former is 22″, and for the latter 2′ 54″; ſo the true altitudes were 64° 2′ 3″ and 17° 4′ 16″. The difference 46° 57′ 47″, and half difference 23° 28′ 53″, the obliquity of the ecliptic by theſe obſervations.

OBSERVATION.

164. For the ſame year, M. le Monnier, in his Hiſtoire Celeſte, concludes, from the obſervations of *Picard* and others, the obliquity of the ecliptic was 23° 28′ 50″.

OBSERVATION.

165. In 1734, M. Godin went to Marſeilles, and obſerved the obliquity of the ecliptic, 23° 28′ 20″; which was but 4″ leſs than M. Loville made it in 1715: and theſe numbers M. de la Caille adopted for the year 1748.

OBSERVATION.

166. The mean obliquity of the ecliptic for the year 1775, is ſettled by aſtronomers at 23° 28′ 0″; and ſuppoſing it to increaſe in every eight years and an half near 14″, and the next nine years to decreaſe near 22″, they reckon the mean decreaſe of the obliquity of the ecliptic near 8″ in every ſeventeen years and an half, or near 48 hundredths of a ſecond per year.

167. M. de la Lande, in Aſtronomical Tables, ſays 48 (hundredths) per year, and gives the obliquity of the ecliptic for

	°	′	″
1775 —	23	28	0
1776 —	23	28	1
1777 —	23	28	3
1778 —	23	28	5
1779 —	23	28	8
1780 —	23	28	10

168. By the foregoing original obſervations, the obliquity of the ecliptic was, in the time of

	Years.			°	′	″
Pytheas	324	before Chriſt	—	23	28	56
			or	23	27	58
			the medium	23	28	27
Walther	1487	after Chriſt	—	23	29	14
Tycho Brahe	1586	—	—	23	29	14
			or	23	29	21
Wright	1597	—	—	23	28	32
Gaſſendus	1636	—	—	23	28	51
Ricciolus	1646	—	—	23	29	24
Caſſini	1655	—	—	23	29	15
Mouton	1601	—	—	23	29	35
			or	23	28	33
			the medium	23	29	4
Richer	1672	—	—	23	28	48
La Hire	1681	—	—	23	28	51
Wertzelbaur	1686	—	—	23	28	53
Picard	1686	—	—	23	28	50
Loville	1715	—	—	23	28	24
Godin	1734	—	—	23	28	20
La Caille	1748	—	—	23	28	20

169. From theſe numbers it appears that, was the obſervation of Pytheas ſet wholly aſide, and the mean times and obſervations of Walther, Tycho, and Wright, compared with the obliquity of the ecliptic for the year 1748, the obliquity for 1557 was 23° 29′ 1″, and its decreaſe was 41″ in 191 years, or 21 hundredths of a ſecond per year. If Bernard Walther's numbers be followed, it is but 20 hundredths. If Tycho Brahe's numbers are followed, it is 33 hundredths per year. But if Edward Wright's numbers are followed, make the moſt of it that can be made, and it is but 16 hundredths of a ſecond per year.

170. The refractions applied in deducing the foregoing concluſions were a medium of thoſe by Dr. Halley, Sir Iſaac Newton, Mr. Flamſtead, and Dr. Brook Taylor, publiſhed in Dodſon's Calculator, or Collection of Mathematical Tables. Theſe are different from the Abbé de la Caille's, and Dr. Bradley's, as may appear from the refractions given

given by thoſe eminent aſtronomers, for the altitudes 15°, 30°, 45°, and 60°, only.

	Alt. 15°	Alt. 30°	Alt. 45°	Alt. 60°
	′ ″	′ ″	′ ″	″
La Caille	3 49 ——	1 54 ——	1 6 ——	38
Bradley	3 30 ——	1 38 ——	57 ——	33
Halley	3 17 ——	1 32 ——	54 ——	31
Newton	3 16 ——	1 31 ——	53 ——	30
Flamſtead	3 0 ——	1 23 ——	48 ——	29
Taylor	3 20 ——	1 33 ——	54 ——	31

171. By the foregoing obſervations, it may be concluded that, if the obliquity of the ecliptic has been decreaſing for 100 years paſt, the decreaſe has not amounted to more than 20″ in that 100 years, to the year 1748. If Edward Wright's obſervation be compared, that decreaſe doth not exceed 8″. And greater differences than this we find in the refractions by different authors, for the altitude of 30°, and between that elevation and the horizon.

172. In determining the obliquity of the ecliptic by the tropical altitudes in the temperate zone, the leaſt tropical altitude, in ſome of the foregoing original obſervations, may be ſuppoſed to have been uncertain to 20″. And about half that error may be ſuppoſed to have been introduced, if the greateſt tropical altitude be applied, and the obſerved elevation of the pole ſtar, both above and below the pole. By Sir Iſaac Newton's table of refraction, the difference between the ſummer and winter refraction will be as follows:

	Summer.		Winter.	
	°	′	″	′ ″
Altitudes	5 ——	8	40 ——	9 46
	10 ——	4	36 ——	5 10
	15 ——	3	4 ——	3 28
	20 ——	2	17 ——	2 33
	25 ——	1	46 ——	2 0
	30 ——	1	26 ——	1 36

And greater differences ariſe, by comparing the refractions obſerved near the horizon, in the temperate zone, and at the polar circle, as derived from the obſervations of

of the academicians, who went there to meaſure a degree of latitude, in 1736. Thus,

			Temp. Zone.		Polar Circle.
Altitudes	° ′	Refractions	′ ″		′ ″
———	1 25	———	20 1	———	24 11
———	2 0	———	17 26	———	20 3
———	2 30	———	15 17	———	18 30
———	3 37	———	12 7	———	14 3

173. In the temperate zone, the difference between the ſummer and winter refraction is nearly between an eighth and a ninth part of the whole. At the altitude of 30°, that difference amounts to 10″; and therefore it is reaſonable to conclude, that the variable ſtate of the refractions, and the want of knowing them more perfectly, for the times and places of the former obſervations, may have contributed towards ſome of theſe irregularities. But from the whole it appears, that there is no probability of the ecliptic altering its obliquity a minute of a degree, for 280 years to come.

174. Theſe concluſions are drawn from actual obſervations. Beſide theſe, there were ſome other aſtronomers of great eminence, from the time of Walther to De la Hire, who made the obliquity of the ecliptic but little more than 23° 28′; theſe were,

175. OBSERVATION.

		°	′	″
In 1450	Purbachius — — —	23	28	0
1514	Vernerus — — —	23	28	0
1536	Copernicus — — —	23	28	0
1609	Maginus — — —	23	28	2
1670	Mengoli — — —	23	28	24

and ſeveral others; who were men not eaſily to be deceived by the admiſſion of erroneous refractions and parallaxes.

176. I have been the more particular in producing and comparing theſe valuable obſervations of antiquity, becauſe the modern aſtronomers have declared the obliquity of the ecliptic to be ſo leſſening, that in 175000 years the ecliptic and equator will nearly coincide; that the annual decreaſe affects

affects the aſtronomical tables; and that this change muſt neceſſarily ariſe from the theory of gravitation. Let one or two of theſe decrements be compared with the real obſervations, to ſee how nearly they agree.

177. From the year 1600 to 1756 is 156 years. Profeſſor Mayer of Gottingen gives 1′ 12″ for the mean diminution of the obliquity of the ecliptic in that time. This, added to 23° 28′ 7″ the mean obliquity for 1756 gives 23° 29′ 19″ for the obliquity 1600, which is 47″ more than it was by Edward Wright's obſervation, near that time.

178. Tycho's obſervation agrees nearly with Mayer's allowance for the yearly diminution.

179. From 1487 to 1756 is 269 years. Mayer gives 2′ 4″ for this time, which added to 23° 28′ 7″, gives 23° 30′ 11″ the obliquity for 1487, which is 57″ more than it was at that time by Walther's obſervation.

180. The ſeven laſt obſervations by Richer, De la Hire, Wertzelbaur, Picard, Loville, Godin, and La Caille, are not antient enough to draw any concluſions from them. One of Mouton's obſervations, thoſe by Caſſini and Ricciolus, are too great for an agreement with the tables; and that by Gaſſendus agrees nearly with the diminution allowed by the tables.

181. Finally, whilſt the Engliſh aſtronomers make the obliquity of the ecliptic for the year 1775, from 23° 27′ 58″ $\frac{5}{10}$ to 23° 27′ 59″ $\frac{8}{10}$; the French aſtronomers make it, from 23° 27′ 48″ $\frac{6}{10}$ to 23° 27′ 49″ $\frac{1}{10}$, which is 10″ difference.

182. In 1764, M. Le Monier, at Paris, had in his obſervatory a large and moſt accurate braſs mural arch, made by a no leſs ingenious inſtrument-maker than the Engliſh and Gottingen arches were made by, with a name on it too deep to be eraſed or altered. Nevertheleſs, we now find by comparing the publications of the two nations, that the Engliſh and French differ 10″ in the obliquity of the ecliptic, which is 20″ on the diſtance of the tropics, or difference between the tropical altitudes. Notwithſtanding this, we are told every year, in England, of the exact agreement of the Engliſh and Gottingen obſervations; whilſt the French, who know perfectly well how to uſe and diſtinguiſh the beſt inſtruments, by their publications, ſhew us there is now a dif-

a difference of 20″ of a degree, reſulting from the application of them.

183. Having, by theſe unqueſtionable proofs and authorities, determined that the obliquity of the ecliptic cannot be altering at the rate which is given out by the moſt celebrated modern aſtronomers; I might here inſert the names of thoſe antient aſtronomers who have ſettled that obliquity different from what we now find it, and ſhew the fallacies which lead them to thoſe erroneous concluſions. But, as this would be a digreſſion, it may be neceſſary only to obſerve, that, as they attributed erroneous numbers not only to the refractions, but to the ſun's horizontal parallax, which latter is now known to be inconſiderable, the errors which they fell into, could not but naturally follow.

184. From the time of Pytheas to Eratoſthenes, Hipparchus, and Ptolemy, we find they reckoned the obliquity of the ecliptic from 23° 52′ 40″ to 23° 51′ 20″, then Albategnius made it 23° 35′. Afterwards there was a ſudden fall; for, from the time of Regiomontanus, Bernard Walther, Tycho Brahe, Kepler, Gaſſendus, Ricciolus, Hevelius, and Mouton, they acknowledged that their original obſervations would make the obliquity of the ecliptic but little more than 23° 30′.

185. In comparing the preſent obſervations with thoſe made at ſuch diſtant periods of time, there can be no fallacy. If any law of diminution be aſſerted by the modern aſtronomers, and that diminution be true, it muſt compare with thoſe ancient obſervations; otherwiſe it may be juſtly taken for error, and derived from wrong principles, whatever ſpecious pretences may be in their favour. Nor is it to be ſuppoſed, conſidering the uncertainty of the ſtate of refraction, that Tycho Brahe's obſervations, although they agree with the ſuppoſed diminution, are to be admitted before Edward Wright's, Bernard Walther's, or any of the others.

186. To aſſert the continual diminution of the obliquity of the ecliptic, is to ſuppoſe the Author of Nature has deſigned, that when the ecliptic and equator do coincide, which they undoubtedly will, if ſuch a continued diminution there be, the northern regions will become as unfit for human habitation through cold, as the torrid zone through heat; and thereby the parts of the habitable world become much reduced; although the human ſpecies, by a courſe of generation

generation do naturally increaſe in number and multiply. A ſuppoſition which can hardly be admitted to ſuit with the deſign of the Creator, who has ſhewn infinite wiſdom in the framing of the world, and infinite power and goodneſs in the continuance of it in the ſame order as it was firſt made.

187. The agreement which there is in the foregoing concluſions ſeems to make it a doubt, whether there has been any diminution in the obliquity of the ecliptic for 300 years paſt. The concluſions drawn from obſervations made at the diſtance of ſhort periods of time depend wholly on the knowledge of the ſtate of the air, as indicated by the barometer and thermometer. Here the differences are very ſmall from one year to another, and therefore it is difficult to draw inferences from ſuch obſervations; nor is it altogether certain that the barometer and thermometer may indicate the refractions accurately. For it has been already ſuſpected by certain philoſophers, that the elaſticity and denſity of the atmoſphere itſelf may not, in the groſs, be without a continual change; or, that the refractive power thereof may not remain the ſame without ſome ſmall alteration at the end of long periods of time.

188. The ſun's place in the ecliptic may be nearly enough found to ſhew his ſituation amongſt the fixed ſtars, by help of the chart of the fixed ſtars; but for finding the exact declination of the ſun for any day, or part of a day, recourſe muſt be had to the tables.

189. In this chart, the ecliptic is the apparent path of the ſun; it is divided into twelve ſigns, Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, Piſces, each ſign containing thirty degrees; and the mark or character of the ſign placed at the entrance into, or beginning of each ſign, and reckoning from the right hand towards the left, which is the order of the ſigns in the heavens.

190. For purpoſes requiring no great accuracy, in determining the ſun's place in the ecliptic, every Biſſextile, or leap year, it may be ſuppoſed that the ſun enters into the correſpondent ſigns of the ecliptic, for the months of November and December, between the 21ſt and 22d days of the month; for the month of March, between the 20th and 21ſt; for the month of April, between the 19th and

20th; for May and June, between the 20th and 21ſt; and for July, Auguſt, September, and October, between the 22d and 23d days of the month; and for each of the three following years after Biſſextile, or leap year, nearly a quarter of a day ſooner. The Biſſextile year is known by dividing the year by 4, and, if nothing remains, it is Biſſextile or leap, if any thing remains, it ſhews the number of years after. The intermediate degrees are eaſily known, by comparing the number of days from the entrance into the ſigns, with the number 30, the degrees in each ſign, which makes it nearly a degree per day. But this is for no correct purpoſes.

191. The ſun's longitude is the number of ſigns, degrees, and minutes, it is diſtant from Aries; and that part of the equator coming to the meridian of any place, with the ſun, ſhews the ſun's right aſcenſion in degrees and minutes, beginning to reckon where the ecliptic and equator cut each other.

192. The equator in the heavens is a great circle, every part of which is 90 degrees diſtant from the north and ſouth poles in the heavens. The equator begins to be reckoned at the ſame place as the ecliptic begins at, and is numbered from 0 degrees to 360 degrees quite round the heavens.

193. As the heavens do appear to move round the axis of the earth produced once in 24 hours, or rather ſhort thereof by $3' 56''$ of time nearly; it is evident, that ſome point of the equator muſt be on the meridian of a place of obſervation, when the ſun is on the meridian of that place. And as many degrees, minutes, and ſeconds, or hours, minutes, and ſeconds of time, as that point is from the beginning of the equator, this is the right aſcenſion of the ſun at that time.

194. The number of hours, minutes, and ſeconds of time, which the ſun appears to riſe or ſet before or after ſix o'clock, is called the aſcenſional difference. This is found by calculation, having the latitude of the place and the ecliptic place of the ſun, or his declination, and from the aſcenſional difference are found the lengths of the days at the different times of the year.

195. The moon appears always in ſome part of the zodiac, but apparently moves from the right hand towards the left, or from weſt towards the eaſt, in the ſpace of a day 13 degrees at a medium; ſometimes more, at other times leſs.

The

The planets Mercury, Venus, Mars, Jupiter, and Saturn, always appear within this zodiac or zone.

196. By this chart or map, the ſtars may be known; for when the ſun's place in the ecliptic is found, if it be before noon, count as many hours as it is before noon to the right of the ſun's place; but if it is paſt noon, count as many hours as it is paſt noon to the left of the ſun's place; and oppoſite to the latitude, whether north or ſouth, in the angle of meeting, is the ſtar, if any; or point over your head, or in the zenith at that time, and hence the north and ſouth, eaſt and weſt points of the horizon may be nearly judged of on the chart, with the intermediate points, and the ſituation of the fixed ſtars to the zenith of any place be nearly judged of, in order to acquire a knowledge of the principal. Fixed ſtars, particularly ſuch as are uſed in making obſervations for the latitudes of places, on land or at ſea, and the zodiacal ſtars uſed in finding the longitude at ſea by help of the moon. When two of theſe charts are joined together in length, 180° may be counted from any ſign or degree of the ecliptic either eaſtward or weſtward. And when two of them are joined in breadth, the ſituation of the ſtars may be ſeen, which are under the elevated pole.

197. As all the fixed ſtars have a particular apparent motion parallel with the ecliptic, at the rate of 50 ſeconds of a degree per year in longitude, whereby in a long ſeries of time they would appear to move quite round the poles of the ecliptic, it follows that their declinations, which are their neareſt diſtances from the equator, muſt be continually altering, ſome increaſing in declination, and others decreaſing. For this reaſon, it has been found requiſite to correct both the declinations and right aſcenſions of the fixed ſtars, before they can be uſed in geography or navigation, by thoſe little differences whereby they either gain or loſe in declination, and gain in right aſcenſion; and the right aſcenſion and declination of 300 of thoſe fixed ſtars, I have corrected to the beginning of the year 1780, as in the following table.

198. A correct Catalogue of the right aſcenſions and declinations of 300 fixed ſtars in the northern and ſouthern hemiſpheres, with their annual variation of right aſcenſion in ſeconds and decimal parts of a ſecond of time; and their annual variation of declination in ſeconds and decimal parts of a ſecond of a degree, whether additive or ſubtractive; the whole being deduced from the moſt authentic aſtronomical obſervations, and corrected to the beginning of the year 1780.

By S. DUNN,

Teacher of Mathematics, LONDON.

In this Catalogue, the annual right aſcenſions are ſub tractive before the beginning of 1780, afterwards additive. The declinations marked additive are ſubtractive before the beginning of 1780, afterwards they are additive. And the declinations marked ſubtractive, are additive before 1780, afterwards ſubtractive. The Greek letter of Bayer is put at the left hand; the figure following it denotes the magnitude of the ſtar; then follows the name of the conſtellation to which the ſtar belongs, next is the right aſcenſion in time, with its annual variation; and laſtly, the declination, with its annual variation. But the Pole ſtar belongs to the conſtellation Urſa minor; Capella to Hircus; Procyon to Canis minor; Regulus to Leo; Arcturus to Bootes; Rigel to Orion; Sirius to Canis major; Spica to Virgo; Antares to Scorpio; and Fomahaut to Piſces.

Stars of the Northern Hemiſphere.

January 1, 1780.			R^t Aſcenſ. h ′ ″	An. Var. Add. ″	North. Declination. ° ′ ″	An. Var. Add. ″
γ	2	Pegaſus	0 1 56	3,08	13 57 58	20,04
δ	3	Andromeda	0 27 36	3,16	29 39 18	20,01
α	3	Caſſiopea	0 28 7	3,30	55 19 38	20,00
γ	3	Caſſiopea	0 43 34	3,49	59 31 10	19,70
α	2	*Pole Star*	0 47 44	10,05	88 7 50	19,40
β	2	Andromeda	0 57 27	3,29	34 26 59	19,43
δ	3	Caſſiopea	1 11 20	3,71	59 4 59	19,07
ε	3	Caſſiopea	1 38 46	4,13	62 34 29	18,27
α	4	Triangle	1 40 39	3,49	28 30 5	18,19
γ	4	Aries	1 41 29	3,27	18 12 39	18,16
β	3	Aries	1 42 30	3,28	19 43 38	18,12
γ	2	Andromeda	1 50 28	3,61	41 15 56	17,82
α	3	Piſces	1 50 41	3,09	1 41 44	17,80
α	3	Aries	1 54 48	3,34	22 24 59	17,63
β	4	Triangle	1 56 29	3,48	33 56 15	17,56
γ	4	Triangle	2 4 18	3,52	32 49 15	17,22
γ	3	Cetus	2 31 56	3,12	2 18 7	15,87
γ	3	Perſeus	2 49 0	4,24	52 37 43	14,96
α	2	Cetus	2 50 48	3,13	3 12 57	14,81
β	2	Meduſa	2 53 55	3,84	40 5 37	14,64
α	2	Perſeus	3 8 43	4,19	49 3 39	13,74
δ	3	Perſeus	3 27 19	4,17	47 3 57	12,52
η	3	Pleiades	3 34 26	3,54	23 24 36	12,01
ζ	3	Perſeus	3 40 20	3,72	31 12 51	11,60
ε	3	Perſeus	3 43 8	3,94	38 21 24	11,41
γ	3	Hyades	4 7 18	3,39	15 4 55	9,59
δ	4	Hyades	4 10 16	3,44	17 0 44	9,35
ε	3	Taurus	4 15 48	3,48	18 40 42	8,93
α	1	Taurus	4 23 19	3,43	16 3 13	8,33
α	1	*Capella*	5 0 28	4,40	45 45 17	5,33
β	2	Taurus	5 12 24	3,78	28 24 15	4,29
γ	2	Orion	5 13 21	3,22	6 8 2	4,19
ζ	3	Taurus	5 24 31	3,58	20 59 30	3,24
β	3	Auriga	5 43 24	4,41	44 54 6	1,64
α	1	Orion	5 43 17	3,25	7 28 11	1,60

January 1, 1780.			Rt Ascens. h ′ ″	An. Var. Add. ″	North. Declination. ° ′ ″	An. Var. Add. ″
θ 3		Auriga	5 44 44	4,08	37 10 37	1,51
η 4		Caſtor	6 9 37	3,63	21 33 16	0,02 Subtract.
μ 4		Pollux	6 9 45	3,63	22 30 57	0,07
γ 3		Pollux	6 25 1	3,48	16 34 16	2,02
ε 3		Caſtor	6 30 24	3,71	15 19 55	2,49
ζ 3		Caſtor	6 51 3	3,58	20 52 37	4,27
δ 3		Pollux	7 6 58	3,61	22 23 20	5,61
β 3		Canis minor	7 15 13	3,27	8 43 19	6,32
α 2		Caſtor	7 20 32	3,87	32 21 12	6,73
α 1		*Procyon*	7 27 48	3,21	5 47 2	7,35
β 2		Pollux	7 31 52	3,75	28 32 33	7,66
β 4		Cancer	8 4 31	3,15	9 51 0	10,23
γ 4		Cancer	8 30 33	3,52	22 14 56	12,11
δ 4		Cancer	8 32 10	3,44	18 57 16	12,22
ι 3		Urſa major	8 41 5	4,25	48 53 29	13,00
α 3		Cancer	8 43 49	3,31	12 42 3	13,01
κ 4		Urſa major	8 48 32	4,20	48 0 37	13,30
θ 3		Urſa major	9 18 6	4,22	52 40 9	15,11
ο 4		Leo	9 29 24	3,24	10 33 3	15,79
ε 3		Leo	9 33 20	3,45	24 16 35	15,99
μ 3		Leo	9 40 13	3,47	27 1 58	16,33
η 3		Leo	9 55 20	3,31	17 49 46	17,07
α 1		*Regulus*	9 56 39	3,24	13 2 10	17,13
ζ 3		Leo	10 4 24	3,34	24 30 20	17,47
γ 3		Leo	10 7 4	3,31	20 57 1	17,61
ρ 4		Leo	10 21 33	3,18	10 26 2	18,14
β 2		Urſa major	10 48 27	3,75	57 33 21	19,01
α 2		Urſa major	10 50 0	3,89	62 56 2	19,05
δ 3		Leo	11 2 23	3,22	21 43 48	19,37
θ 3		Leo	11 2 41	3,18	16 37 51	19,38
β 2		Leo	11 37 51	3,12	15 48 9	19,93
β 3		Virgo	11 39 13	3,08	3 0 22	19,94
γ 2		Urſa major	11 42 9	3,24	54 55 1	19,98
δ 2		Urſa major	12 4 27	3,05	58 15 18	20,04
η 3		Virgo	12 8 39	3,07	0 33 30	20,02
ε 2		Urſa major	12 44 17	2,69	59 9 14	19,69
δ 3		Virgo	12 44 33	3,06	4 45 52	19,69
	3	Cor Car.	12 45 43	2,86	39 30 36	19,66

January 1, 1780.			R^t Aſcenſ. h	′	″	An. Var. Add. ″	North. Declination. o	′	″	An. Var. Subt. ″
ε	3	Virgo	12	51	17	3,01	12	28	21	19,57
ζ	2	Urſa major	13	15	2	2,45	56	4	42	19,01
ζ	3	Virgo	13	23	31	3,08	0	31	58	18,77
η	2	Urſa major	13	38	52	2,41	50	24	58	18,24
η	3	Bootes	13	44	13	2,88	19	30	43	18,06
α	3	Draco	13	58	23	1,50	65	25	49	17,45
α	1	*Arcturus*	14	5	41	2,82	20	21	4	17,16
γ	3	Bootes	14	23	13	2,43	38	16	10	16,31
ζ	3	Bootes	14	30	39	2,86	14	40	54	15,93
ε	3	Bootes	14	35	28	2,63	28	0	37	15,67
β	3	Bootes	14	53	41	2,28	41	15	57	14,62
δ	3	Bootes	15	6	46	2,44	34	8	46	13,82
ι	4	Draco	15	20	4	1,31	59	44	27	12,91
δ	3	Serpens	15	24	19	2,87	11	17	12	12,68
α	2	Corona	15	25	9	2,05	27	28	0	12,60
α	3	Serpens	15	33	30	3,01	7	7	49	12,06
β	3	Serpens	15	36	2	2,76	16	7	21	11,87
ε	4	Serpens	15	39	52	2,98	5	9	8	11,62
γ	3	Serpens	15	46	18	2,75	14	24	7	11,14
θ	4	Draco	15	57	48	1,14	59	9	10	10,23
γ	3	Hercules	16	12	14	2,66	19	40	54	9,18
β	3	Hercules	16	20	48	2,59	21	59	52	8,51
η	4	Draco	16	21	3	0,78	62	0	5	8,41
ζ	3	Hercules	16	33	1	2,30	32	0	34	7,51
η	4	Hercules	16	35	22	2,07	39	21	7	7,31
ε	3	Hercules	16	51	52	2,29	31	15	43	5,96
α	2	Hercules	17	4	37	2,74	14	39	17	4,91
δ	3	Hercules	17	7	0	2,47	25	6	44	4,69
α	2	Serpentar.	17	24	44	2,78	12	44	13	3,17
β	3	Draco	17	25	29	1,36	52	28	13	3,06
β	3	Serpentar.	17	32	37	2,97	4	40	25	2,52
γ	3	Serpentar.	17	36	53	3,01	2	48	24	2,15
μ	4	Hercules	17	37	53	2,38	27	52	3	2,03
θ	3	Hercules	17	48	44	2,06	37	17	24	1,08
γ	3	Draco	17	51	54	2,21	51	31	14	0,80
										Add.
α	1	Lyra	18	29	30	2,06	38	35	14	2,48
β	3	Lyra	18	41	58	2,22	33	7	10	3,55

January 1, 1780.			Rt Aſcenſ. h ′ ″	An. Var. Add. ″	North. Declination. ° ′ ″	An. Var. Add. ″
θ	4	Serpentar.	18 45 17	3,00	3 56 0	3,80
δ	3	Lyra	18 46 50	2,11	36 37 48	3,97
ε	4	Aquila	18 49 39	2,73	14 47 2	4,18
γ	3	Lyra	18 51 4	2,26	32 23 58	4,29
ζ	4	Aquila	18 55 16	2,69	13 33 3	4,67
δ	3	Draco	19 12 28	0,06	67 16 24	6,22
δ	3	Aquila	19 14 25	3,02	2 41 26	6,26
β	3	Cygnus	19 21 51	2,43	27 30 31	6,90
α	4	Sagitta	19 30 17	2,69	17 31 16	7,58
γ	3	Aquila	19 35 31	2,28	10 5 23	8,02
δ	3	Cygnus	19 46 12	1,86	44 36 6	8,24
α	2	Aquila	19 40 30	2,91	8 17 56	8,36
η	4	Antinous	19 41 16	3,07	0 27 19	8,45
β	3	Aquila	19 44 32	3,02	5 52 32	8,71
γ	3	Cygnus	20 14 20	2,16	39 13 43	11,00
ε	4	Delphinus	20 22 43	2,88	10 34 8	11,58
ζ	4	Delphinus	20 25 2	2,82	13 55 44	11,74
β	3	Delphinus	20 27 15	2,82	13 50 30	11,91
α	3	Delphinus	20 29 25	2,79	15 8 53	12,05
δ	4	Delphinus	20 33 12	2,81	14 17 46	12,32
α	2	Cygnus	20 33 57	2,05	44 30 7	12,39
γ	4	Delphinus	20 36 28	2,79	15 20 39	12,54
ε	3	Cygnus	20 37 17	2,40	33 9 9	12,62
ζ	4	Cygnus	21 3 25	2,55	29 20 0	14,31
α	4	Equiculus	21 4 49	3,01	4 21 19	14,37
ε	4	Pegaſus	21 11 54	2,79	18 52 18	14,80
α	3	Cepheus	21 13 17	1,43	61 39 23	14,92
β	4	Cepheus	21 25 43	0,81	69 35 44	15,64
ε	3	Pegaſus	21 33 22	2,95	8 52 30	16,00
μ	4	Cygnus	21 34 18	2,64	27 45 26	16,05
ζ	3	Pegaſus	22 30 29	2,99	9 41 19	18,46
η	3	Pegaſus	22 32 42	2,80	29 4 31	18,55
ο	4	Andromeda	22 51 49	2,72	41 8 55	19,12
β	2	Pegaſus	22 53 8	2,89	26 53 27	19,15
α	2	Pegaſus	22 53 50	2,99	14 1 31	19,17
γ	4	Cepheus	23 30 27	2,31	76 24 15	19,86
α	2	Andromeda	23 57 23	3,06	27 52 33	20,04
β	3	Caſſiopea	23 57 51	3,04	57 56 8	20,04

January 1, 1780.			Rt Aſcenſ. h ′ ″	An. Var. Add. ″	South. Declination. ° ′ ″	An. Var. Subt. ″
β	3	Hydrus	0 13 34	2,72	78 29 50	20,01
α	2	Phœnix	0 15 23	3,01	43 36 23	20,00
β	2	Cetus	0 32 32	3,01	19 11 56	19,85
β	3	Phœnix	0 56 15	2,73	47 55 57	19,46
η	4	Cetus	0 57 31	3,00	11 21 5	19,44
θ	4	Cetus	1 13 4	3,01	9 19 17	19,07
γ	3	Phœnix	1 18 49	2,67	44 26 51	18,90
ω	1	Eridanus	1 29 31	2,26	58 21 35	18,56
α	3	Hydrus	1 51 49	1,87	62 38 44	17,73
ο	4	Cetus	2 8 15	3,03	3 58 56	17,05
δ	3	Cetus	2 28 15	3,08	0 37 48	16,07
ε	3	Cetus	2 28 57	2,90	12 48 51	16,03
θ	3	Eridanus	2 49 57	2,30	41 11 39	14,85
ζ	3	Cetus	3 5 10	2,91	9 38 53	13,93
ε	3	Eridanus	3 22 37	2,89	10 12 42	12,79
δ	3	Eridanus	3 32 45	2,88	10 31 29	12,10
γ	3	Eridanus	3 47 57	2,79	14 8 51	11,02
α	3	Rhomboid	4 11 40	0,74	61 1 42	9,15
α	3	Xiphias	4 29 16	1,28	55 30 19	7,78
β	3	Eridanus	4 57 3	2,95	5 23 3	5,56
β	1	*Rigel*	5 4 0	2,89	8 28 8	4,97
η	3	Orion	5 13 26	3,02	2 36 52	4,17
β	4	Lepus	5 18 51	2,58	20 56 52	3,69
δ	2	Orion	5 20 48	3,07	0 28 37	3,54
α	3	Lepus	5 23 3	2,65	17 59 39	3,34
ε	2	Orion	5 25 4	3,05	1 21 26	3,17
ζ	2	Orion	5 29 41	3,04	2 4 20	2,77
α	2	Columba	5 31 43	2,20	34 12 6	2,56
β	4	Xiphias	5 31 34	0,50	62 38 15	2,48
γ	4	Lepus	5 55 19	2,53	22 31 51	2,26
κ	3	Orion	5 37 21	2,85	9 45 35	2,10
δ	4	Lepus	5 41 52	2,57	20 54 26	1,68
β	3	Columba	5 43 13	2,11	35 51 54	1,55
						Add.
ζ	3	Canis major	6 11 53	2,31	29 58 40	0,93
β	3	Canis major	6 13 1	2,63	17 51 42	1,92
α	1	Canopus	6 19 5	1,34	52 34 57	1,60
ν	3	Argo navis	6 31 2	1,84	43 0 44	2,62
α	1	*Sirius*	6 35 29	2,69	16 25 6	2,97

January 1, 1780.			Rt Aſcenſ. h	′	″	An. Var. Add. ″	South. Declination. °	′	″	An. Var. Add. ″
ε	3	Canis major	6	49	59	2,36	28	41	7	4,23
δ	2	Canis major	6	59	28	2,45	26	3	27	5,03
π	3	Argo navis	7	9	23	2,13	36	42	41	5,88
η	2	Canis major	7	15	24	2,39	28	53	11	6,37
σ	3	Argo navis	7	22	17	1,94	42	51	54	6,96
ζ	2	Argo navis	7	55	52	2,12	39	23	28	9,62
γ	2	Argo navis	8	2	47	1,85	46	41	40	10,16
ε	3	Argo navis	8	17	59	1,26	58	48	32	11,30
δ	2	Argo navis	8	38	37	1,61	53	54	24	12,73
λ	3	Argo navis	8	58	55	2,21	42	33	7	14,09
β	1	Argo navis	9	10	45	0,75	68	48	51	14,79
ι	3	Argo navis	9	11	14	1,65	58	21	32	14,79
κ	3	Argo navis	9	15	20	1,86	54	4	31	15,03
α	2	Hydrus	9	16	48	2,96	7	42	45	15,08
υ	3	Argo navis	9	41	36	1,50	64	3	20	16,45
θ	3	Argo navis	10	35	10	2,12	63	14	44	18,64
η	2	Argo navis	10	36	34	2,27	58	32	0	18,68
μ	3	Argo navis	10	37	22	2,55	48	15	39	18,81
α	4	Crater	10	49	6	2,95	17	8	1	19,05
δ	3	Centaurus	11	56	59	3,05	49	29	41	20,03
α	4	Corvus	11	57	6	3,07	23	30	7	20,03
ε	4	Corvus	11	58	51	3,06	21	23	46	20,04
δ	3	Croſero	12	3	35	3,10	57	31	31	20,04
γ	3	Corvus	12	4	32	3,09	16	19	34	20,04
α	1	Croſero	12	14	33	3,22	61	52	48	20,01
δ	4	Corvus	12	18	31	3,09	15	17	18	19,98
γ	2	Croſero	12	19	35	3,36	70	54	53	19,97
β	3	Corvus	12	22	52	3,14	22	10	38	19,95
α	4	Muſca	12	24	18	3,42	67	56	15	19,94
γ	2	Centaurus	12	29	30	3,27	47	44	50	19,89
γ	3	Virgo	12	30	33	3,08	0	14	18	19,88
β	4	Muſca	12	33	1	3,52	66	54	1	19,84
β	2	Croſero	12	35	2	3,42	58	29	3	19,83
θ	4	Virgo	12	58	35	3,09	4	21	30	19,41
γ	3	Hydrus	13	7	0	3,22	22	0	21	19,22
ι	3	Centaurus	13	8	18	3,34	35	32	43	19,20
α	1	*Spica*	13	13	38	3,15	10	0	23	19,06
ε	3	Centaurus	13	26	5	3,69	52	20	16	18,70
ζ	3	Centaurus	13	41	55	3,66	46	11	43	18,15

January 1, 1780.		Rt Aſcenſ. h ′ ″	An Var. Add. ″	South. Declination. ° ′ ″	An. Var. Add. ″
β 2	Centaurus	13 48 30	4,09	59 17 59	17,91
θ 3	Centaurus	13 53 45	3,52	35 16 26	17,69
κ 4	Virgo	14 1 12	3,19	9 9 25	17,37
λ 4	Virgo	14 7 14	3,22	12 21 5	17,20
η 3	Centaurus	14 21 38	3,75	41 10 42	16,43
α 3	Pyxis	14 25 0	4,68	63 59 56	16,26
α 1	Centaurus	14 25 2	4,41	59 55 17	16,26
α 3	Lupus	14 27 24	3,89	46 25 44	16,14
α 2	Libra	14 38 44	3,30	15 6 55	15,30
β 3	Lupus	14 44 12	3,86	42 13 51	15,22
κ 3	Centaurus	14 44 56	3,84	41 12 21	15,17
γ 4	Scorpio	14 51 15	3,48	24 24 15	14,70
γ 3	Triangle	14 58 39	5,29	67 50 44	14,40
β 2	Libra	15 5 14	3,30	8 33 29	13,94
γ 3	Lupus	15 20 32	3,94	40 24 34	12,97
γ 4	Libra	15 23 15	3,34	14 2 33	12,77
β 3	Triangle	15 35 55	5,06	62 43 34	11,96
ρ 4	Scorpio	15 43 22	3,68	28 33 17	11,38
π 4	Scorpio	15 45 35	3,60	25 27 58	11,32
δ 3	Scorpio	15 47 21	3,52	21 58 50	11,09
β 2	Scorpio	15 52 41	3,50	19 11 16	10,69
ν 4	Scorpio	15 58 44	3,47	18 52 27	10,24
δ 3	Serpentarius	16 2 50	3,14	3 6 44	9,92
ε 3	Serpentarius	16 6 42	3,16	4 8 25	9,63
σ 4	Scorpio	16 7 42	3,62	25 2 50	9,56
α 1	*Antares*	16 15 58	3,66	25 55 36	8,93
τ 4	Scorpio	16 22 13	3,70	27 24 28	8,44
α 3	Triangle	16 25 33	6,16	68 35 32	8,26
ζ 3	Serpentarius	16 25 4	3,30	10 6 22	8,20
ε 3	Scorpio	16 35 58	3,90	33 52 21	7,31
μ 3	Scorpio	16 37 1	4,04	37 38 57	7,26
ζ 3	Scorpio	16 39 9	4,20	41 57 37	7,09
η 3	Scorpio	16 56 26	4,28	42 55 29	5,66
η 2	Serpentarius	16 57 47	3,44	15 26 14	5,52
θ 3	Serpentarius	17 8 11	3,67	24 45 39	4,61
α 3	Ara	17 14 52	4,61	49 40 34	4,12
υ 4	Scorpio	17 15 51	4,09	37 5 56	4,01
λ 3	Scorpio	17 18 42	4,08	36 55 22	3,77
θ 3	Scorpio	17 21 33	4,30	42 50 10	3,53

January 1, 1780.			Rt Aſcenſ. h	′	″	An. Var. Add. ″	South. Declination. °	′	″	An. Var. Add. ″
κ	3	Scorpio	17	27	17	4,13	38	53	46	3,03
ι	3	Scorpio	17	32	13	4,18	40	1	7	2,60
ζ	4	Serpens	17	48	53	3,16	3	39	30	1,11
γ	4	Sagittarius	17	51	41	3,87	30	24	21	0,89
μ	4	Sagittarius	18	0	37	3,60	21	6	0	0,10
										Subtract
δ	3	Sagittarius	18	6	54	3,85	29	54	5	0,43
ε	3	Sagittarius	18	9	35	4,00	34	27	57	0,67
λ	3	Sagittarius	18	14	24	3,72	25	31	25	1,09
φ	4	Sagittarius	18	31	55	3,77	27	11	54	2,61
σ	3	Sagittarius	18	41	37	3,74	26	33	8	3,45
ζ	3	Sagittarius	18	48	36	3,84	30	10	34	4,06
ο	4	Sagittarius	18	51	29	3,60	22	2	51	4,31
τ	4	Sagittarius	18	53	12	3,77	27	58	17	4,45
π	3	Sagittarius	18	56	11	3,59	21	21	28	4,70
β	4	Sagittarius	19	6	47	4,34	44	50	55	5,57
α	4	Sagittarius	19	8	37	4,20	41	0	30	5,73
ι	4	Antinous	19	25	55	4,01	1	45	37	7,17
δ	4	Pavo	19	46	54	5,92	66	42	27	8,78
α	3	Capricornus	20	5	27	3,35	13	12	53	10,30
α	2	Pavo	20	8	8	4,89	57	25	14	10,44
β	3	Capricornus	20	8	38	3,39	15	27	46	10,54
α	3	Indus	20	22	1	4,29	48	2	30	11,48
β	3	Pavo	20	24	55	5,65	66	58	19	11,64
γ	4	Pavo	21	8	2	5,28	66	21	5	14,50
β	3	Aquarius	21	19	59	3,18	6	39	42	15,35
γ	3	Capricornus	21	27	49	3,35	17	38	52	15,66
δ	3	Capricornus	21	34	53	3,33	17	6	47	16,06
γ	3	Grus	21	40	33	3,70	38	23	18	16,35
α	2	Grus	21	54	20	3,98	48	0	51	17,00
α	3	Aquarius	21	54	29	3,10	1	22	55	17,03
α	3	Toucan	22	3	17	4,31	61	20	52	17,40
γ	3	Aquarius	22	10	18	3,09	2	29	22	17,71
β	3	Grus	22	29	27	3,68	48	5	38	18,43
λ	4	Aquarius	22	41	9	3,16	8	44	44	18,82
δ	3	Aquarius	22	42	58	3,22	16	59	14	18,87
α	1	*Fomahaut*	22	45	17	3,34	30	46	56	18,94
φ	4	Aquarius	23	2	56	3,13	7	13	50	19,38

A Chart of the Zodiacal Stars Used in finding the Longitude at Sea by the Moon. By S. Dunn, Teacher of the Mathematical Sciences London.

Published with Improvements by S. Dunn. No 6. Clements Inn, February 1st 1774 Price Coloured. 6d 2.6d

Lat.d

80 70 60 50 40 30 20 10 10 20 30 40 50 60 70 80

Lat.d

γ Pegasi α Pegasi α Cygni α Lyræ β Draco. α Herculis α Aquilæ

23 22 21 20 19 18 17

ECLIPTIC

Feb.y Jan.y Dec.r

β Capricorni Fomalhaut Antares

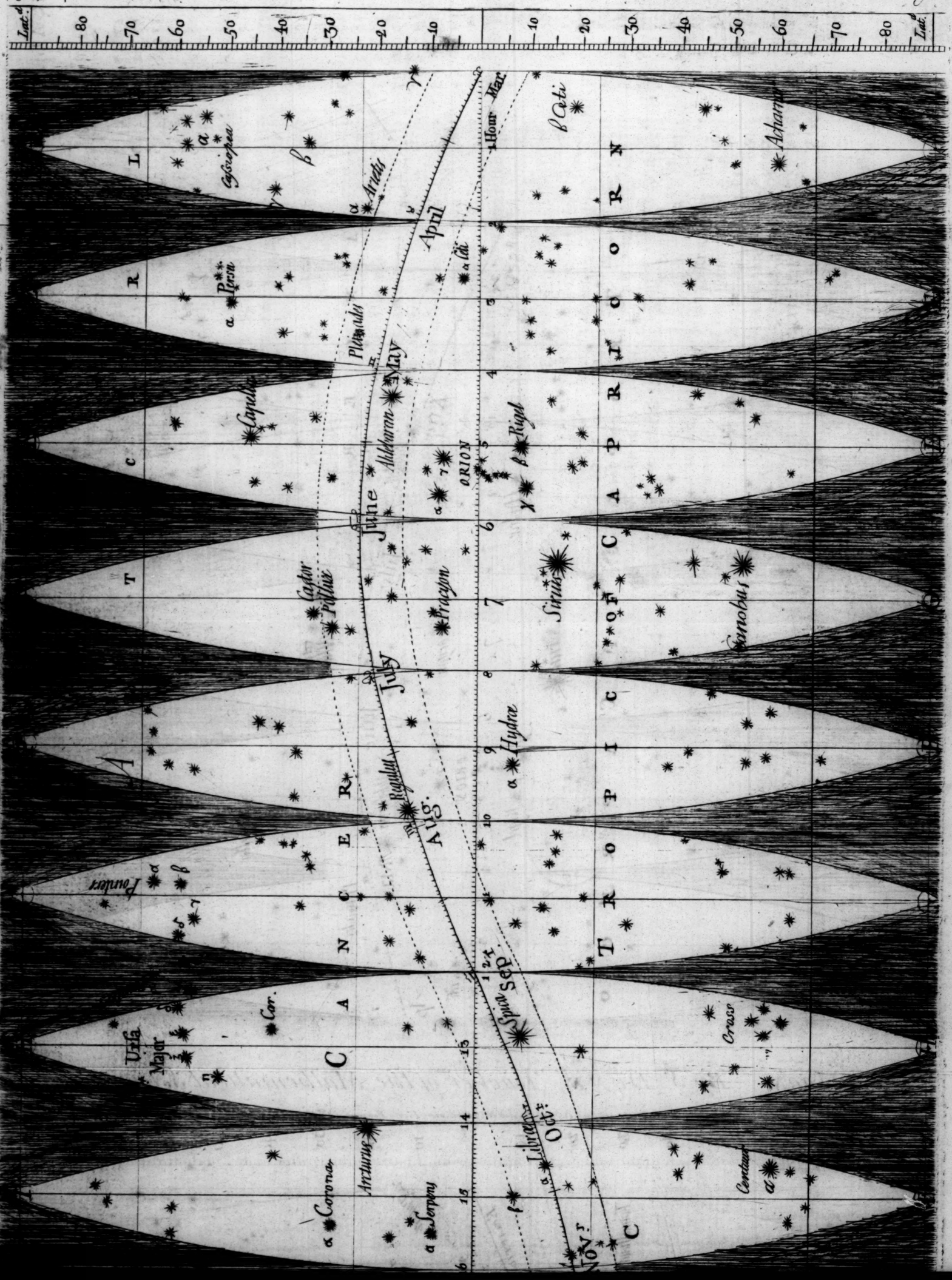
TROPIC OF

EQUATOR

ZODIAC

ANTARCTIC CIRC

Shews the Sun, Moon, & Planets Places in the Zodiac, by an Ephemeris, & their Position to the Seasons Stars with other Phenomena. Shews Hours of a Day past Noon counting to the Left, for before Noon to the Right of the Sun, gives the Stars on the Meridian



199. By this table of the ſtars, and the lines put up after the manner already deſcribed, it will be very eaſy to determine the rate of going of a clock, watch, or other time-keeper, independent of any obſervation of the ſun. Thus, for inſtance, obſerve when any one of the ſtars in the catalogue is on the meridian by the lines; and then ſet the clock to twelve, or to any other hour, or hour and minute, and let it go on either one, two, or three days, or more; and then obſerve, by the ſame ſtar, what hour, minute, and ſecond of time by the clock the ſtar is on the meridian; ſo the gain or loſs of the clock in that interval will be known. And, if it loſes $3'\ 56''$ nearly of the ſtar per day, the clock may then be ſaid to keep time nearly agreeing with the motion of the ſun; but to ſet it to ſuch mean ſolar time, the clock muſt have the index of its dial-plate put to twelve, when the ſun is on the meridian. Or, if it is to be ſet to mean ſolar time, when the ſun is on the meridian, the index of the dial-plate muſt be put as much before or after twelve, as the equation of time for that day indicates.

200. The ſame method is to be obſerved, in determining how much a time-keeper gains or loſes in any part of a day, from one inſtant of time to another, during any interval. But in this caſe a proportional part muſt be calculated for, as it is leſs than an whole day. And by the ſame method may a clock be ſet to mean ſolar time, or to ſolar time itſelf, by obſerving the meridian tranſit of a fixed ſtar; by taking the difference between the right aſcenſion of the ſun for any day, and the right aſcenſion of the ſtar; and the clock is to be put ſo much before or after the ſtar's tranſit over the meridian, to be nearly with ſolar time. But if the clock is to be ſet to mean ſolar time, it muſt be as much before or after ſolar time as the equation table indicates for that day.

201. In the making of obſervations, aſtronomers uſually omit all conſiderations concerning the figures of the conſtellations. The conſtellations of the heavens, in which the fixed ſtars are ſuppoſed to be ſituated, are of very great antiquity. But, in the making of aſtronomical obſervations, the configurations of the conſtellations are beſt laid aſide, and the names of the ſtars more eaſily learnt and underſtood by help of the Greek letters introduced by Bayer.

202. The

202. The configurations of the conſtellations in the northern and ſouthern hemiſpheres, exhibit the vicinity of many of the conſtellations to one another, and ſhew with what extreme difficulty the ſtars are found in the heavens by a verbal account of their ſituation in any parts of the conſtellations only; whilſt the method of finding them otherwiſe is moſt facile, and attended with no ambiguity.

203. Having given the precepts for determining the latitudes of places by the ſun, but reſerved the ſolar tables and their uſe to another place; and having given the declinations of the fixed ſtars; I ſhall here give a few from a great number of obſervations made for verifying the quadrant before deſcribed, as paſted accurately on a board, and mounted againſt a fixed wall, in a proper ſituation for taking meridian altitudes of the ſun and ſtars.

OBSERVATION.

204. 1774, February 17th, at noon.

	°	′	″
Sun's co. altitude obſerved — —	63	20	0
Another reading — — —	63	18	0
The medium is — — —	63	19	0
Refraction add to co. altitude — —	0	1	48
True zenith diſtance — —	63	20	48
Sun's declination — — —	11	50	30
Latitude of the place — — —	51	30	18

In this obſervation the ſun was biſected horizontally.

OBSERVATION.

205. The ſame day, February 17th, evening.

Rigel obſerved on the meridian.

	°	′	″
The zenith diſtance obſerved — —	59	58	0
The refraction add to co. altitude —	0	1	32
True zenith diſtance — — —	59	59	32
Rigel's declination ſouth — —	8	28	38
Latitude of the place — —	51	30	54

OBSER-

OBSERVATION.

206. The ſame evening, February 17th.
Sirius obſerved on the meridian.

	°	′	″
The zenith diſtance obſerved — —	67	53	0
The refraction add to co. altitude —	0	2	18
True zenith diſtance — —	67	55	18
Sirius's declination ſouth — —	16	24	48
Latitude of the place — —	51	30	30

OBSERVATION.

207. 1774, February 18th, at noon.
The ſun's upper limb obſerved.

	°	′	″
The zenith diſtance obſerved — —	62	42	0
The refraction add — — —	0	1	45
True zenith diſtance — — —	62	43	45
Sun's ſemidiameter add — —	0	16	13
Sun's true zenith diſtance — —	62	59	58
Sun's declination ſouth — —	11	29	16
Latitude of the place — —	51	30	42

208. From theſe and other obſervations it might be proved, that this inſtrument, ſo eaſily formed, and having its errors, if any, expunged, is ſufficiently correct, when uſed with care, for many geographical purpoſes; for the place where I made theſe obſervations is near Temple Bar, in London, whoſe latitude is 51° 30′ 40″ very nearly; and the medium of the three foregoing obſervations determines the latitude as correctly as could be done by almoſt any other inſtrument.

209. There are various inſtruments made uſe of for determining the latitudes of places on land and at ſea; the beſt of which, to be uſed on land, require to be ſteadily fixed and well-adjuſted before they are applied, and then they will perform

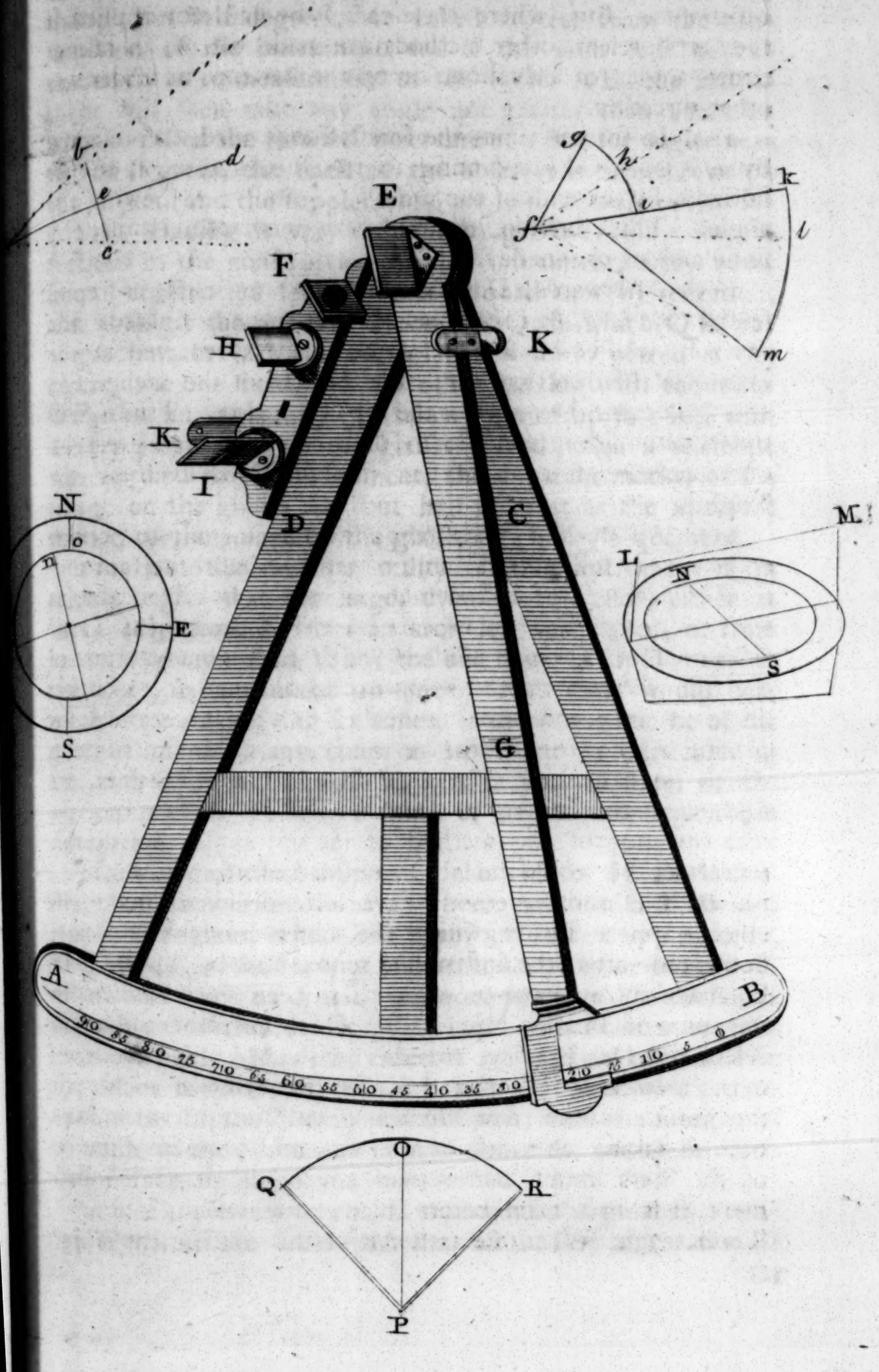
perform well. Such are the largeſt and beſt aſtronomical quadrants. But, where theſe cannot be had, nor applied if they are preſent, other methods are made uſe of, in taking either angles of elevation, or on the horizon, or under any other direction.

210. In former times the foreſtaff was uſed, repreſented by *a*, *b*, *e*, *c*; *a*, *b*, pointing to the ſun, and *a*, *c*, to the horizon, by the ends of the croſs *b*, *c*; and *a*, *e*, ſhewing the angle. This method, though very incorrect, is uſed by ſome nations to this day.

211. Afterwards, Davis's quadrant was uſed, as repreſented by *f*, *g*, *h*, *i*, *k*, *l*, *m*; *l*, *f*, pointing towards the horizon, whilſt the ſhadow of the ſun was received at the centre in the direction *h*, *f*; and the ſum of the arches *h*, *i*, and *k*, *l*, gave the angle of the ſun's elevation. This inſtrument, although preferable to the former, had no great accuracy, although it be not yet wholly diſuſed even by the Engliſh.

212. About the year 1730, ſeveral ingenious perſons endeavoured to find out ſome better and more certain method of taking angles, and ſuch as would be applicable at ſea. Amongſt theſe it appears by the Philoſophical Tranſactions, that Thomas Godfrey of Philadelphia was one of the firſt who attempted to reform the old method of taking angles at ſea; and this he effected in a great meaſure, by placing a reflecting mirror at the centre of an inſtrument, which he called the Mariner's Bow. Theſe accounts, we find, were tranſmitted to England; and if he was not the real inventor of the method of taking angles by double reflection, his could be no inconſiderable ſtep towards it; for both of thoſe methods have a tendency to make the object appear ſteady whilſt the obſervation is making. Some time after, the inſtrument repreſented by A, B, C, D, E, made its appearance, which has been ſince known by the name of Hadley's quadrant; which inſtrument the late celebrated Dr. Bradley, together with Mr. Hadley, tried in the months of Auguſt and September, lying at anchor, in the mouth of the river Medway; and found it to anſwer, both in taking altitudes of the ſun, and angular diſtances of the ſtars, much better than any other manual inſtrument that had been before uſed; for which, ſee Phil. Tranſ. 1732. When the arch AB of this inſtrument is 45°,

to


a
b
c
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90 85 80 75 70 65 60 55 50 45 40 35 30
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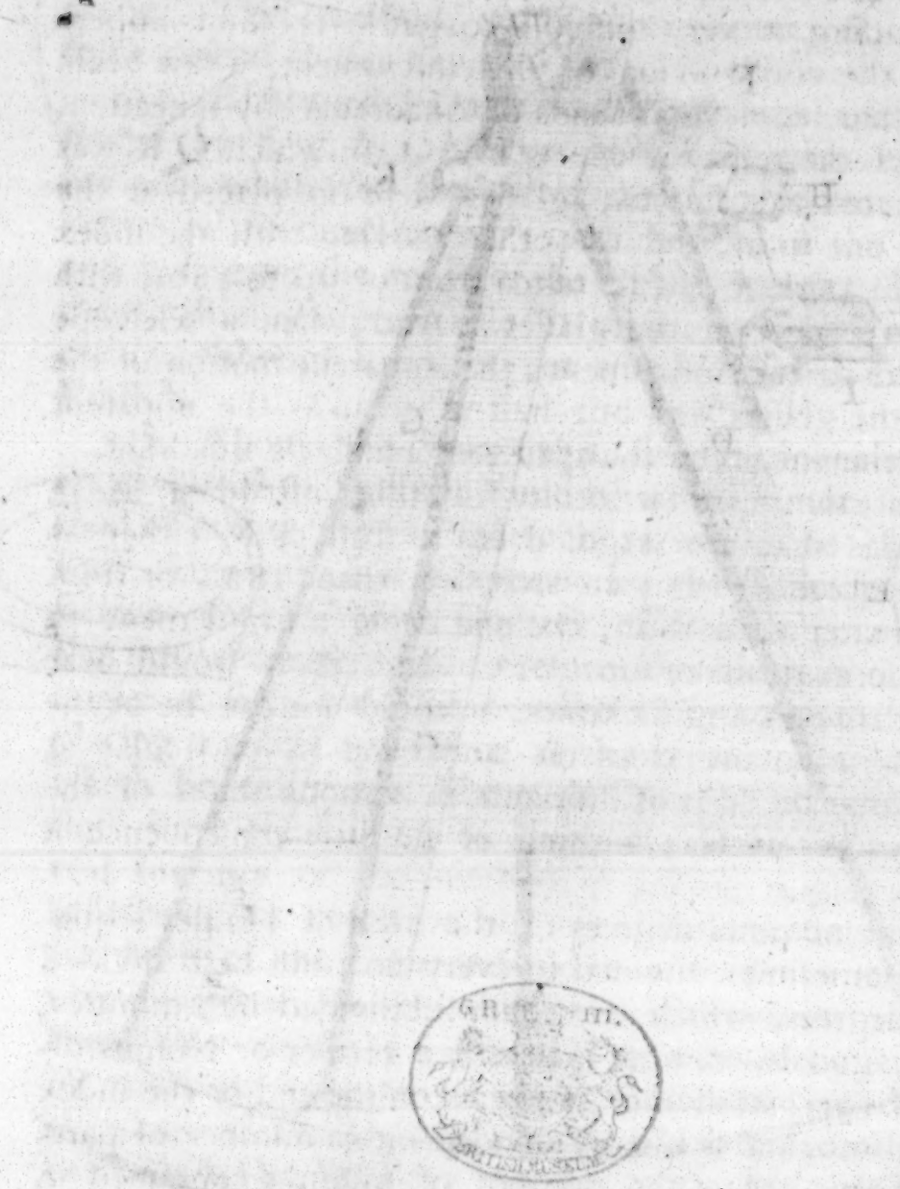


to the radius AE, it is divided into 90 parts, which, although but half degrees, are called degrees from the conſtruction of the inſtrument, and if the arch A B be extended ſo as to meaſure 60° to the radius A E, the inſtrument will then take any angle not greater than 120°, by what is called the forward obſervation. But for angles near to 180 degrees, the back of the obſerver is turned towards the object, and the ſupplemental angle to 180° is taken.

213. Finding it very difficult to get over ſome imperfections in the conſtruction of this inſtrument, a few years ſince I conſtructed ſeveral kinds of quadrants by reflection, and amongſt the reſt was one as P R Q, in which Q R was a quadrant or 90°, and two glaſſes were ſo placed at the centre, the one fixed, and the other moving with the index O P, that an angle might be taken from 0° up to 180°, with a very large view on the glaſſes. And when a teleſcope was applied to the inſtrument, the apparent motion of the image on the glaſſes was but half as great as the apparent motion of the image on the glaſſes of Hadley's quadrant.

214. But the peculiar utility of this inſtrument is in taking angles that are large, even to full 180°, either as ſuppoſedly formed from an arch leſs than 180°, or from an arch greater than 180°; the one being a ſupplement to the other, ſo as to make up 360°. And as it would bear a teleſcope magnifying ſix times, it could not but be of the greateſt uſe in many caſes on land, and in particular at ſea, in finding the dip of horizon in various ſtates of the air and weather at ſea; a thing of no little conſequence in navigation.

215. The nominal degrees of the arch of Hadley's quadrant are ſometimes divided to every 20′, but beſt divided into half degrees, which are again ſubdivided into minutes, ither by diagonals, or, what is better, a vernier or Nonius diviſion. The vernier diviſion is placed on the end of the index G, at the limb, and is uſually divided into a number of parts greater by unity than the number of minutes contained in the leſſer diviſions of the limb, and the lines which meet each other on the Nonius and the arch, ſhew the number of minutes on the Nonius, over and above the next leſſer diviſion on the arch.

216. In this inſtrument, E repreſents the great mirror or ſpeculum, H the little ſpeculum for the fore obſervation, F

the dark glaſs between the two ſpeculums for the fore obſervation, and K the eye piece, with its holes or ſights for the fore obſervation.

217. The principles on which the adjuſtment of the inſtrument are founded, are as follows: 1ſt, The great mirror ſhould ſcrew down at right angles to the plane of the inſtrument A B C E D, by means of its cock and ſcrews. 2d, The little mirror ſhould ſtand and move round in a perpendicular poſition to the plane of the inſtrument. 3d, The hole in the eye-piece K ſhould be the ſame diſtance from the plane of the inſtrument, as the place on the little ſpeculum H, where the image of the object is viewed. 4th, One of the objects being ſeen by direct viſion, in the line K H, and the other being brought by reflection from E to H, and from H to K, the obſerver ſees them united, and the diſtance of the index from B the beginning of the quadrant or ſextant, ſhews the angular diſtance of the two objects obſerved.

218. Before the inſtrument is uſed, it ſhould be well adjuſted. This is uſually done by help of a ſhort bar or lever on the back of the quadrant, which moves the little mirror H; ſo that, when the index is ſet to the beginning of the degrees at B, one part of an object, not nearer than a mile, can be ſeen through the unquickſilvered part of the little ſpeculum, whilſt the other part of the object, as ſeen on the quickſilvered part of the ſame ſpeculum, perfects the object. This adjuſtment is uſually made holding the quadrant upright, and proves that the two ſpeculums are parallel to each other, when they are both perpendicular to the place of the quadrant.

219. It may happen that the cock of the great ſpeculum will not ſcrew down at right angles to the plane of the inſtrument as before deſcribed; in ſuch caſe the little ſpeculum may be firſt adjuſted by moving it to a proper ſituation, and there adjuſting it ſo that you can ſee through the eye-hole its image, at a mark on the little ſpeculum, as far diſtant from the plane of the inſtrument as the eye hole is. Then the great ſpeculum may be adjuſted to the little one, as the little one was before adjuſted to the great one.

220. As the adjuſtment of this inſtrument is of the greateſt conſequence, there cannot be too much care taken about it. It is my opinion, that the want of this, and the want of

ingenuity

ingenuity and good abilities in the perſons who have many times uſed it, have been the cauſe why the ſucceſs has not been as has been wiſhed for. Nor is it eaſy to conceive, how any perſon ſhould be inſtantly able to manage ſuch an inſtrument with that certainty that is requiſite in many caſes, when men of no ſmall mechanical abilities have been a conſiderable time coming to a tolerable degree of perfection in applying it.

221. The grand queſtion concerning the truth and certainty of this inſtrument is, whether it will or will not take an angle to that degree of exactneſs, that is requiſite for the purpoſes to which it is wanted to be applied? This has occaſioned many diſputes amongſt practical aſtronomers; and whilſt one party are ready to grant that an angle may be taken by this inſtrument without an error of a minute of a degree, the other party can hardly grant that, in ſome caſes, an angle may be taken without an error of five minutes of a degree.

222. The moſt advantageous purpoſe to which this inſtrument has been already applied, has been that of determining the place of a ſhip at ſea, both in latitude and longitude. The latitude was attainable ſomewhat near the truth by inſtruments uſed before the Hadley's quadrant was invented; but thoſe inſtruments were inſufficient for taking the angular diſtances of the ſun and moon, or moon and ſtars, with that exactneſs that was wanted for the longitude.

223. The ingenious Abbé de la Caille, in page 31, of his Ephemerides, beginning with 1755, ſuppoſes "the "longitude at ſea cannot be determined by the moon and "the uſe of this inſtrument to a leſs error than two degrees, "let the inſtrument be ever ſo perfect, and the obſerver ever "ſo ingenious." And the merit of this aſtronomer is indiſputable. In that Ephemerides, the learned Abbé publiſhed the method of finding the longitude at ſea, by taking the angular diſtance of the ſun and moon, and the moon and certain zodiacal ſtars, with the neceſſary calculation relative to the ſame.

224. In May 1759, a comet appeared, which I obſerved by taking its diſtance from certain fixed ſtars, living then at Chelſea. Theſe obſervations were publiſhed in the daily papers. The inſtrument with which theſe diſtances were

taken was an octant 18 inches radius, with diagonal ſubdiviſions, and framed of mahogany. This inſtrument I have now by me, and have no reaſon to ſuppoſe it has received any material injury by alteration of the framing. From many of the angular diſtances which were then taken, it appeared that their difference ſcarce ever exceeded two minutes of a degree. This made a gentleman who was then preſent conjecture, that the diſtance of the ſun and moon, or moon and ſtar, might be taken to the like accuracy; and that, if a medium of ſeveral obſervations was admitted, it might be more exact, and become of uſe in obſerving for the longitude at ſea. And this I take to have been the time when the practicability of finding the longitude by the moon gained freſh credit; for, in the year 1761, a voyage was made to the ſouth; ſeveral aſtronomical experiments were made; and, amongſt the reſt, the trial of a quadrant in finding the longitude by the moon: and ſoon after were publiſhed obſervations for the longitude by the diſtances of the ſun and moon, and moon and ſtars, warranted as more exact than the former obſervations by M. de la Caille.

225. Thus the obſervations made concerning an erratic ſtar, which was but imperfectly defined, were no otherwiſe than inſtrumental in ſetting ſeveral perſons at work in good earneſt; and the Hadley's quadrant, which before had been but ſparingly applied to thoſe moſt curious purpoſes, and by but a very few ingenious perſons, was tried to its greateſt perfection; and the inſtrument-makers were conſulted about making it as correct as poſſible.

226. The beſt method of dividing and ſubdividing inſtruments, is now generally admitted to be by continual biſection. This method has been publiſhed, and a premium given for it. But, four years before that publication, I communicated another method of biſection to M. Burton, mathematical inſtrument-maker in the Strand, and to another perſon, by which any arch may be divided as accurately as poſſible. This method is applicable, not only in dividing an arch of a circle, but in dividing a right line into any number of equal parts; and was communicated by me in 1763, in the following words, with proper illuſtrations:

"A general

" A general method, whereby either a right line, or the arch of a circle, may be divided and ſubdivided into any number of equal parts more than two, without any error in thoſe diviſions and ſubdiviſions diſcernible to the eye, or by the help of glaſſes.

" It is well known to aſtronomers, and other practical mathematicians, that the accurate diviſion and ſubdiviſion of a circle into degrees and minutes, is a thing of the greateſt import in the conſtruction of angular inſtruments; and that the inaccuracy, which is too often to be met with in the diviſions and ſubdiviſions of aſtronomical quadrants, nautical quadrants, theodolites for land-ſurveying, and other angular inſtruments, doth render the uſual concluſions, deduced from the application and uſe of thoſe inſtruments, either entirely falſe, or very uncertain.

" This ſubject hath been looked upon in a ſerious manner by eminent aſtronomers and practical mathematicians of different nations, and in particular by the late Dr. Edmund Halley; inaſmuch that, when he cauſed his mural quadrant to be graduated, he deviated from the uſual way, by dividing the quadrant into 96 equal parts, although they contained but 90 degrees, or the quarter of a circle.

" The reaſon why he divided the quadrant into 96 equal parts was, becauſe the number 96 admits of a continual biſection, or halfing, down to the number 3; and becauſe it is found by experience, that a right line, or an arch, may be biſected or divided into two equal parts, without the leaſt diſcernible error, when a pair of dividing compaſſes are made uſe of, whoſe points are well pointed; and the common method of biſection is made uſe of, which is deſcribed in Euclid.

" But a way of triſecting or quinqueſecting a right line, or arch of a circle, to the ſame degree of accuracy as by continual biſection, hath not yet been practiſed, nor known to mathematical inſtrument-makers, as far as I have been able to be informed, on which account, this moſt uſeful piece of artifice, the accurate diviſion of a circle into degrees and minutes, is ſaid to have been attained by but a very few perſons in the mathematical inſtrument making way. And this they perform, ſome by one way and ſome by another, which

which I ſhall not here conſider, but give the problem above propoſed.

" Let A repreſent either a finite right line, or the arch of a circle; and let it be required to divide the ſame into *n* number of equal parts, the letter *n* repreſenting either 3, 5, 7, 11, 13, or any other odd number which cannot be biſected or halfed, and leave no remainder. Put *m* equal to ſuch a number that, it being added to *n*, the ſum will continually be diviſible by 2 down to unity; that is, for inſtance, when *n* is 3, *m* will be 1; when *n* is 5, *m* will be 3; when *n* is 7, *m* will be 1; when *n* is 11, *m* will be 5; when *n* is 13, *m* will be 3, &c. Then divide the given right line, or arch, into *n* number of equal parts by a trial or two, by which the *n*th part will be had near the truth, for a firſt aſſumption, and add *m* times this *n*th part to the given right line, or circular arch. Then, by continual biſection of the ſum of the right line or circular arch A, and the ſuperadded parts, *m* times *n*, get the *n*th part of the line or arch A, more correct than by the firſt aſſumption; aſſume this corrected *n*th part in the room of the firſt aſſumed *n*th part, and proceed therewith in all reſpects as before; and thus in two or three operations, the *n*th part of the given right line, or circular arch, will be found.

" I ſhall give an example of the excellency of this method, in dividing an arch of 60°, ſuppoſing it very large, as in the large aſtronomical quadrant.

" The arch of 60° is determinable, without any error, from the radius of the circle; and the arch of 4°, by a few trials, ſomewhat near the truth; theſe 4° ſomewhat near the truth being added to the 60°, make 64°, which are diviſible by 2, till the arch of 15′ is found. And if, by a trial or two, the arch of 4° be found very near the truth, that error will be divided into ſo many parts in the ſubdiviſion, that no viſible error will appear in any of the parts. By the ſame method, the arch of 32° may be transferred from where 58° are perfect; and this 32° will be diviſible by continual biſection, and the quadrant finiſhed.

" By the ſame method may a right line be divided and ſubdivided." Therefore,

227. In dividing of any braſs quadrant, having drawn the arch, and aſſigned a point where the diviſions are to begin, let the chord of 60° be ſet off from that point, and by a very

a very faint continual diviſion and ſubdiviſion, let the arch of 4 degrees be got as accurately as poſſible. This arch of 4 degrees, being added to the end of the 60 degrees, makes an arch of 64 degrees, which is diviſible by a continual biſection. By this method, the arch of 4 degrees may be had without any ſenſible error; and even that error, ſmall as it is, when added to the 60 degrees, divides into ſo many parts, that it may be ſaid to vaniſh in the ſmaller ſubdiviſions. This method has been proved as a moſt admirable one, for dividing mathematical inſtruments into degrees and half degrees; and the ſubdiviſions are had by the vernier diviſion. The arch of 32° ſubdivides by the ſame method of biſection.

228. As there have been many diſputes concerning the certainty and accuracy of the Hadley's quadrant, I ſhall inſert part of a letter, witten by me in the Public Ledger, January 24, 1769, in defence of the Hadley's quadrant, and the method of finding the longitude at ſea by the moon, in anſwer to one that appeared in the ſame paper the 20th of the ſame month; the writer of that letter, ſigning himſelf Philomath, appearing to have an uncommon ſhare of envy againſt the promoters of the diſcovery of the longitude at ſea by the moon; and treating a worthy friend of mine, together with myſelf, in a very unhandſome manner, ſuppoſing me intereſted in the longitude plan, whilſt I never received any emolument from it.

229. "The writer of a letter, ſigned Philomath, in yeſter-
"day's Ledger, has thrown out ſo many falſities, that it is
"no more than doing yourſelf and the public common
"juſtice, to inſert the following ſhort, but true, ſtate of the
"caſe, relative to the hardſhips which this writer pretends
"the maſters of the royal navy are to be ſubject to by the
"diſcovery and practice of the longitude at ſea.

230. "The ſolution of this grand problem has been al-
"ways thought to be of that conſequence and utility, that
"almoſt every commercial nation has offered a great re-
"ward for the diſcovery; and although it has lain among
"the *deſiderata* ſo many ages, it is at length brought to
"light by the continuation of that grand plan, which was
"firſt laid by Lord Bacon, and afterwards brought to ſuch
"ſurprizing height by the illuſtrious Sir Iſaac Newton.
"Since which time ſeveral eminent aſtronomers have con-

"tinued

" tinued the work, and greatly contributed to the perfect-
" ing thereof, amongſt whom is the preſent Aſtronomer
" Royal.

231. " The Engliſh mariner, when he learns the old
" method of ſailing by log and compaſs, courſe and diſ-
" tance, pays the inſtructor that teaches him two or three
" guineas; and, if he is very dull, it may coſt him four or
" five guineas. * * * * * And when he is ever ſo well
" qualified, and has had experience at ſea ever ſo long, he
" will ſometimes be out in his reckoning at ſea, 100, 200,
" or 300 miles, and ſometimes as many leagues. And
" this will be the caſe with the moſt experienced maſter of
" a ſhip that ever traverſed the ocean, without ſome further
" help, ſuch as the diſcovery of the longitude. * * * * *

232. " But here, Philomath; * * * * may learn the me-
" thod of finding the longitude from the rules and in-
" ſtructions which have been publiſhed. And this he may
" do without the help of a maſter to teach him, if he has
" (as he pretends he has) any ſmattering in aſtronomy;
" and when he has learnt it, I am perſuaded, not one of the
" perſons appointed to examine and certify for him, would
" refuſe to give him a certificate *gratis*, if (as he pretends
" to be) himſelf and family are in indigent circumſtances.
" And if he could not learn it without a maſter, I dare ſay,
" either of thoſe perſons would inſtruct a ſhip's maſter in
" indigent circumſtances gratis, and give him his certificate
" free.

233. " The Abbé de la Caille made ſeveral obſervations
" at ſea, and, computing each of them ſingly, found a dif-
" ference which amounted to a degree of longitude; and
" thence concluded the method uncertain to a degree of
" longitude. But the late Mr. Thomas Simpſon has
" proved, that a mean of ſeveral ſuch obſervations muſt
" fall very near the truth itſelf; and the late Dr. Bradley
" was of the ſame opinion, and theſe are names ſuperior to
" that of the Abbé de la Caille. * * * * *

" VERAX."

Thus much againſt the malevolence and envy expreſſed in that letter. As to the ſubject of granting certificates, I have been always of opinion that, to be able to handle the inſtrument well, and to ſuch purpoſe as the longitude problem requires, a perſon ſhould have judicious inſtruction on the

the ſpot, much care, and a ſufficient time to practiſe. And after all, I have found at different times ſuch diſagreement in the obſervations with this inſtrument, as fully convinces me it is not abſolutely perfect. Others have concluded, that the index cannot be moved forward and backward by ſliding on the limb without bending; and that herefrom the ſpring of the index, as they have called it, has produced an error of ſeveral minutes of a degree. But ſuch a ſuppoſition is too ridiculous to be admitted, although it may have come from tbe better ſort of obſervers, there being no defect in the formation of the index that can poſſibly admit of ſuch an error. As to other impediments or imperfections that may be ſuppoſed to ariſe in the uſe and application of this valuable inſtrument, they are here omitted.

234. The magnetic needle is a very uſeful inſtrument in ſeveral parts of ſcience, particularly in ſurveying and navigation. When this needle is ſuſpended on a fine point, and placed over the true meridian of any place, it will reſt either in a poſition coincident with the true meridian, in which caſe it is ſaid to have no declination or variation; or it will reſt with its north point inclining to the eaſtward of the true meridian, and then it is ſaid to have eaſt variation; or if its north point decline to the weſt of the true meridian, it is then ſaid to have weſt variation.

235. The needle is uſually made to carry a card, whoſe circumference is divided into degrees and points of the compaſs. And therefore, if there are two cards of different diameters ſuch as the declinations, the greater and lowermoſt card repreſenting the viſible horizon, whilſt the uppermoſt one repreſents the points of the horizon as they are derived from the magnetic north and ſouth; the variation of all the points of the magnetic horizon from the true horizon, will be ſhewn by inſpection; ſuch an inſtrument is commonly called a rectifier, and may be made by cutting the ſmall circle round the circumference, and faſtening the leſſer over the greater by a thread and knot through the centres.

236. In the preceding diagram, E. W. N. S. repreſent the eaſt, weſt, north, and ſouth points of the true horizon, *n* the north point of the magnetic horizon, and *n o* the variation, which by the figure is as many degrees and minutes as the arch *n o* meaſures, and north weſtwardly. The courſe

of a ſhip at ſea, is regulated by the way it ſails according to the magnetic card; and therefore the variation of the needle muſt be known, otherwiſe the true courſe of the ſhip cannot be known. This compaſs is called the Steering Compaſs at ſea; and by knowing its variation, the ſouthing of the ſun may be known from which meridian altitudes are taken for inferring the latitude. The like may be uſed for obſervations on land.

237. There is a ſecond kind of compaſs, called the Azimuth Compaſs, repreſented by L M N S; in which N S repreſents the magnetic north and ſouth, or meridian of the card; and L M, a fine thread, or wire, ſtretched from end to end of two upright indexes; which indexes are directed towards the ſun, when an obſervation is made by the naked eye, or when the ſun is intenſe, the ſhadow of the thread being made to paſs through the centre, the poſition of the ſun is thereby known. This obſervation is made previous to a calculation, from which the variation of the needle may be found by one obſervation with the compaſs, when the latitude of the place and the day of the month are known.

EXPERIMENT.

238. By this inſtrument, the ſun's bearing being obſerved when he appears riſing, and in like manner obſerved when he is ſetting, or otherwiſe when he appears at nearly equal heights in the morning and afternoon, the middle between theſe two bearings is the true meridian, from which the variation of the needle may be found either on land or at ſea. But ſuch obſervations will give the variation moſt exact, when they are made neareſt to the longeſt or ſhorteſt days in the year.

EXPERIMENT.

239. The like obſervations may be made, by placing the ſharp end of a pin or needle upright, at the centre of either the large or ſmall ſemi-circular plates beforementioned; when they are properly paſted on board, and the variation of the needle may thereby be found readily on land, and when no better inſtrument is preſent at ſea, the ſame may be placed on a ſteering compaſs, as it is hung on gimbols to take off the motion of the ſhip, and the variation may be found.

240. Was

240. Was the pole ſtar exactly at the pole, it would appear at the ſame point of the heavens during every 24 hours, or whilſt the earth is revolving round its own axis. But the pole ſtar being at the diſtance of 1° 54′ nearly from the pole, it is once in every 24 hours as much above that pole; nearly 6 hours after, the ſame height as the pole, but weſtward of it; 6 hours after that, below the pole; and at the end of the next 6 hours, the ſame height as the pole, but eaſtward of it; hereby deſcribing apparently a circle round the pole, in the ſpace of a day, or rather 3′ 56″ ſhort of a day, and a the diſtance of 1° 54′ nearly from the pole itſelf.

241. Was the pole ſtar's diſtance from the pole always the ſame, it might be more generally applied; but as that is not the caſe, becauſe all the ſtars are continually altering their declinations, either increaſing or decreaſing by a ſmall quantity, and likewiſe their right aſcenſions increaſing; it is requiſite to fix the declination of the pole ſtar, before any application is made of it in drawing a true meridian line on land, or before it is applied in determining the variation of the needle either on land or at ſea.

Observation.

242. By the obſervations of Edward Wright, at London 1594, the pole ſtar's greateſt height above the pole was 54° 24′ 30″, and below the pole 48° 39′ 30″; the refraction to the former is 38″, and to the latter 48″; whence the true altitudes were 54° 23′ 52″, and 48° 38′ 42″; the difference is 5° 45′ 10″, half of which is 2° 52′ 35″, the pole ſtar's diſtance from the pole, in 1594, by theſe obſervations.

Observation.

243. In like manner for the year 1660, Gabriel Mouton, at Lyons, obſerved the pole ſtar's altitudes above and below the pole, which, when cleared of refraction as before mentioned, gives 48° 16′ 23″, and 43° 14′ 33″; the difference is 5° 1′ 50″, and half difference 2° 30′ 55″, the pole ſtar's diſtance from the pole by theſe obſervations.

Observation.

244. In 1689, Mr. Flamſtead, at Greenwich, obſerved the pole ſtar's leaſt meridian altitude 49° 7′ 50″; the refrac-

tion being 45″, the true altitude was 49° 7′ 5″, from which take the latitude 51° 28′ 38″, gives 2° 21′ 33″, the pole ſtar's diſtance from the pole by this obſervation.

Observation.

245. The modern aſtronomers have ſettled the pole ſtar's diſtance from the pole 1° 55′ 24″ for the year 1770. This being compared with the obſervation of Edward Wright, gives the pole ſtar's mean yearly approach towards the pole 19″ and 44 hundredths; by Mouton's obſervations, it is 19″ and 37 hundredths; and by Flamſtead's, it is 19″ and 37 hundredths; and therefore I ſhall take the pole ſtar's mean yearly approach towards the pole to be 19″ and 4 tenths of a ſecond per year.

246. From theſe conſiderations, I conclude that the pole ſtar's mean diſtance from the pole, from the beginning of the year 1775 to the beginning of the year 1786, will be as follows:

	°	′	″		°	′	″
1775	1	53	47	1781	1	51	50
1776	1	53	28	1782	1	51	31
1777	1	53	8	1783	1	51	12
1778	1	52	49	1784	1	50	52
1779	1	52	29	1785	1	50	33
1780	1	52	10	1786	1	50	14

And the pole ſtar's diſtance from the pole for the middle of the year 1780, will be 1° 52′ 0″; from which will ariſe an eaſy method of conſtructing an orthographical delineation, for ſhewing the apparent diurnal orbit of the pole ſtar, and its ſituation to the pole, as in the delineation annexed; the deſcription and uſe of which is as follows:

247. The aſtronomical ſolar day is always ſuppoſed to begin at noon, and to be counted on to 24 hours, the beginning of the next day; theſe hours are numbered from 0 to 24 at the circumference, and lines are drawn from them to the centre.

248. The letters and numbers following them at the outer circumference, expreſs the months and days of thoſe months, when the pole ſtar will be over the pole, as many hours of time as are expreſſed under the months and days.

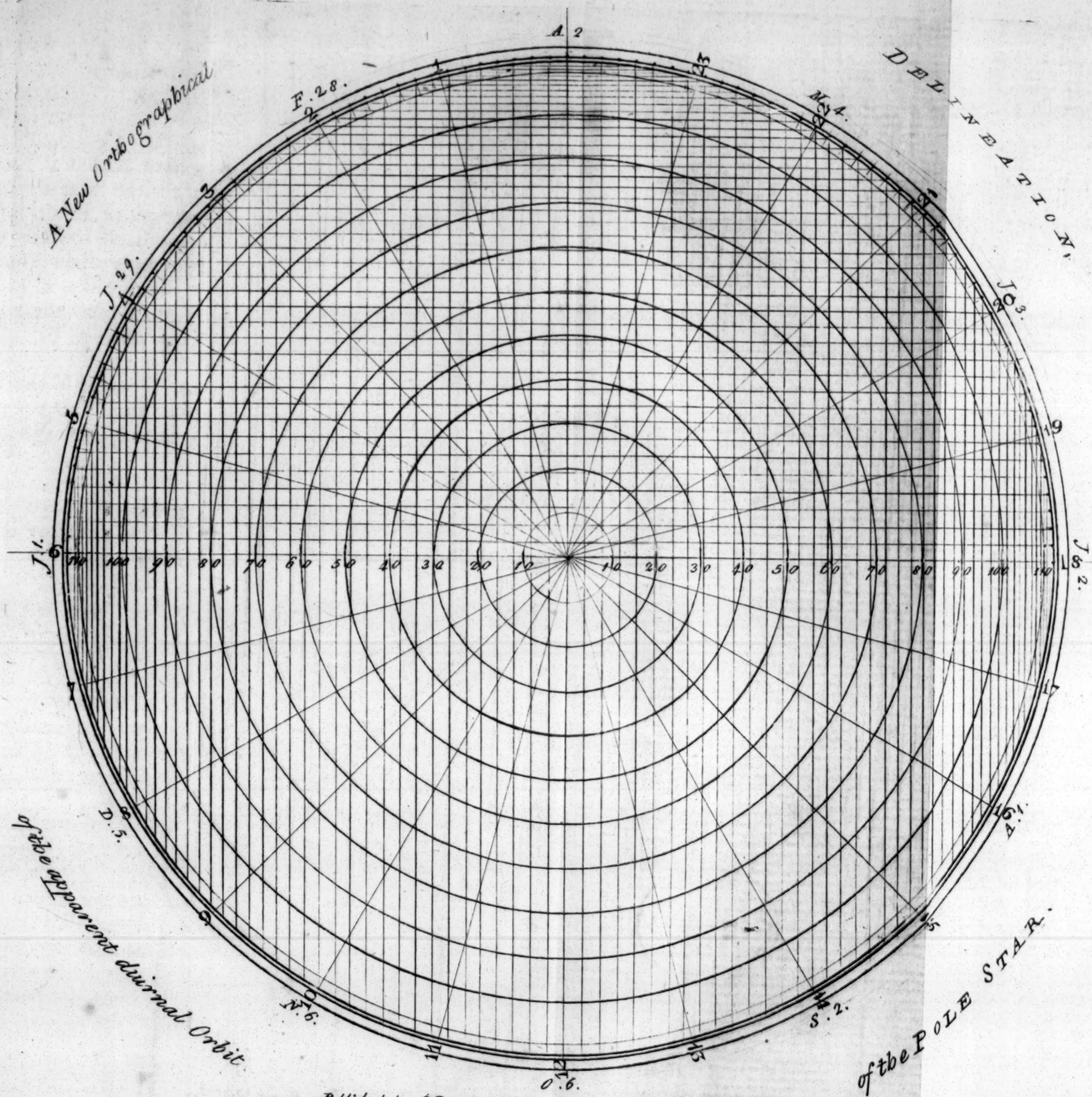
249. The

A New Orthographical

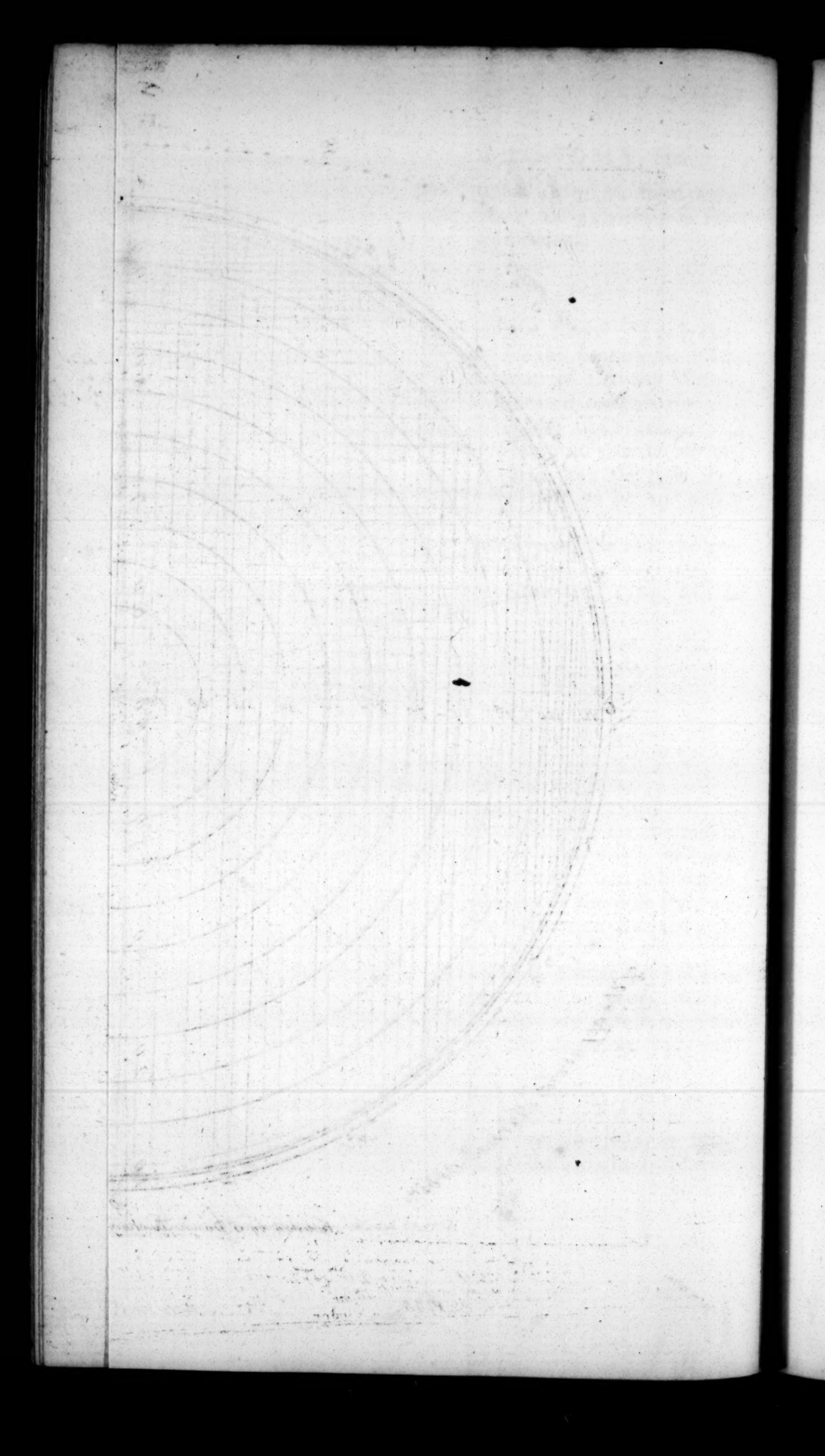
DELINEATION,

of the apparent diurnal Orbit

of the POLE STAR.



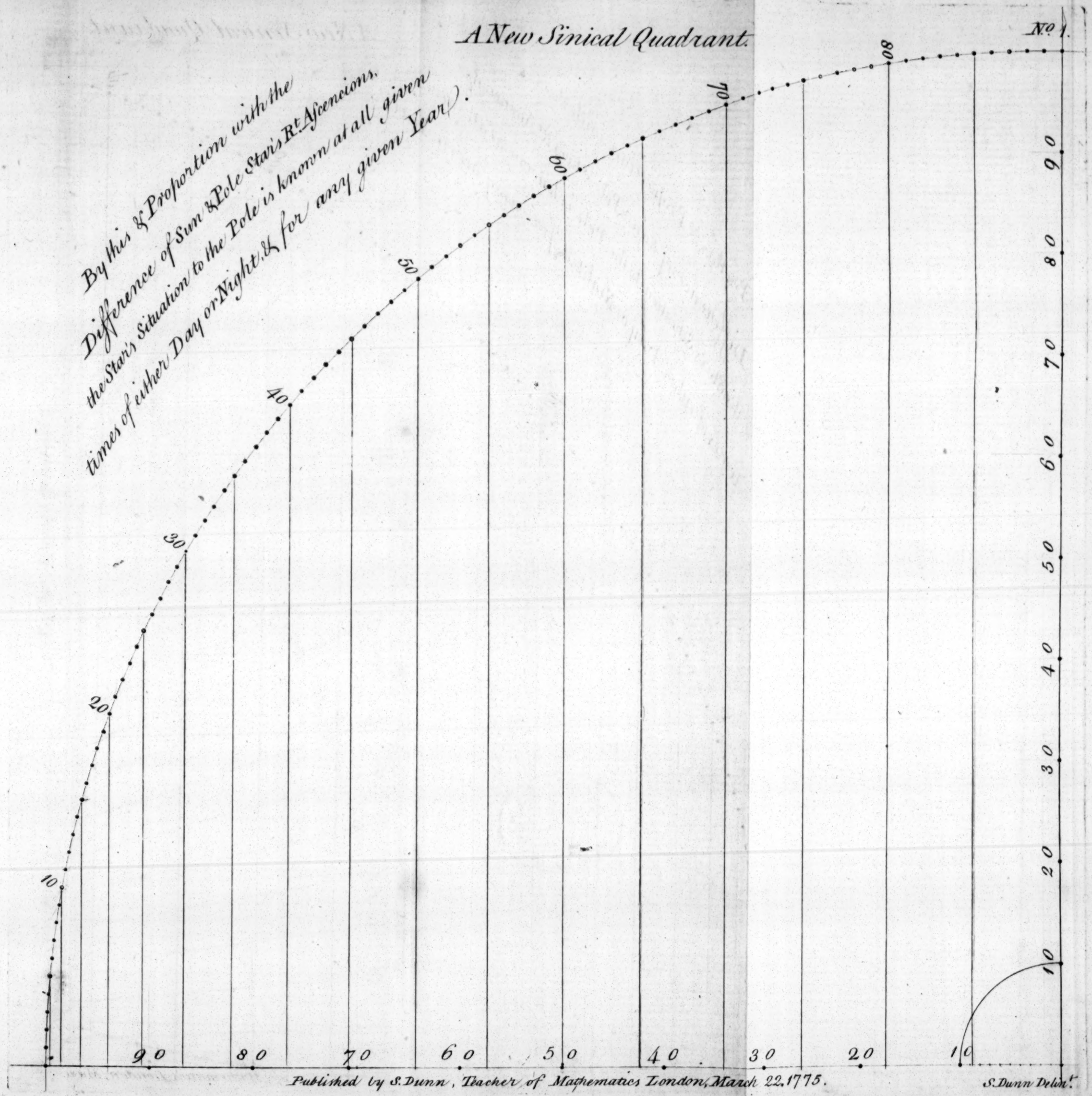
Published by S. Dunn Teacher of the Mathematics Clements Inn March 22. 1774.



A New Sinical Quadrant.

No. 1.

By this & Proportion with the Difference of Sun & Pole Star's Rt. Ascencions, the Star's Situation to the Pole is known at all given times of either Day or Night, & for any given Year).

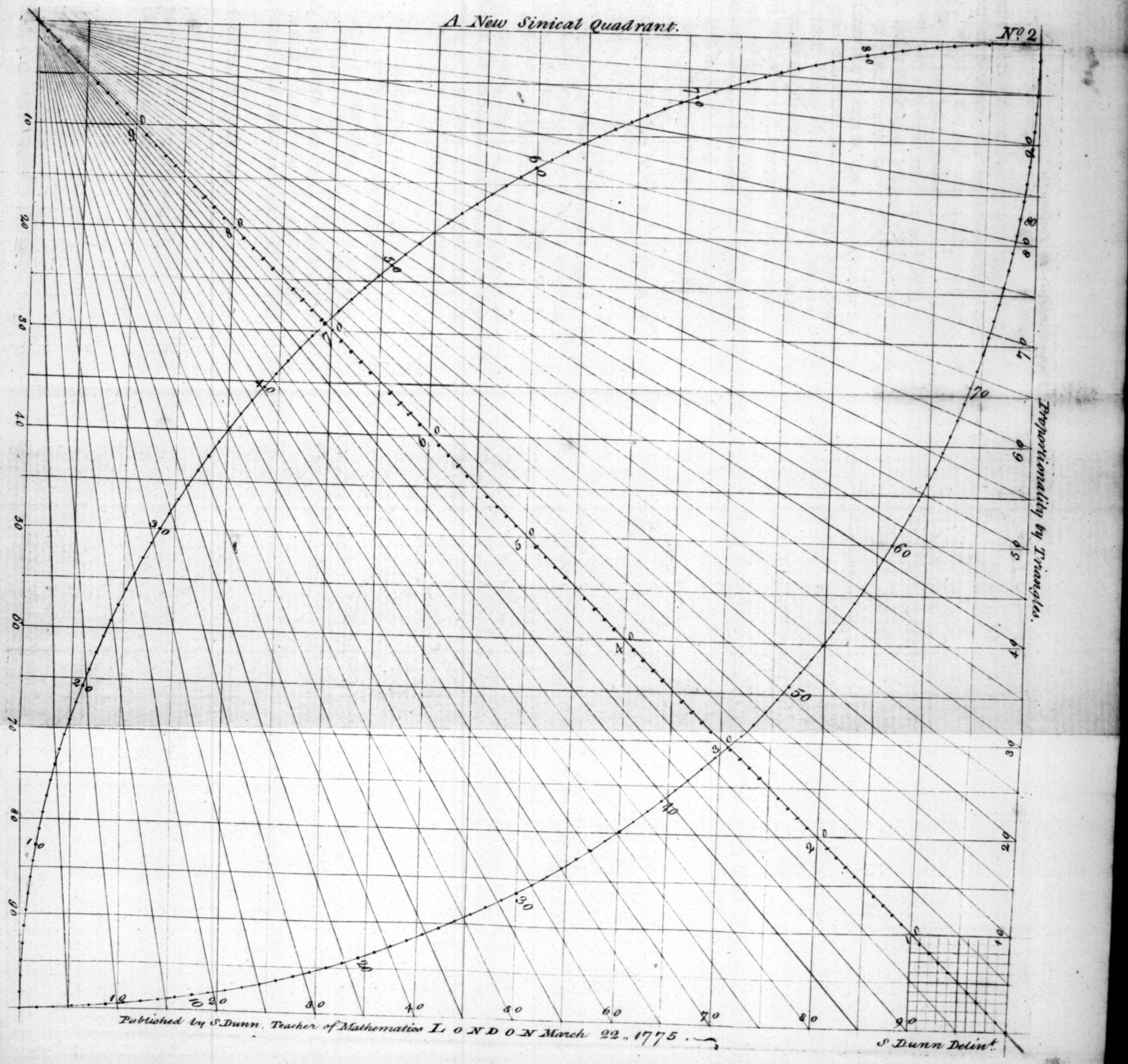


Published by S. Dunn, Teacher of Mathematics London, March 22. 1775.

S. Dunn Delin.

A New Sinical Quadrant.

N.º 2



Published by S. Dunn, Teacher of Mathematics LONDON March 22. 1775.

249. The concentric circles are drawn at the diſtance of every ten minutes from each other; the whole making 112 minutes, or 1° 52′ 0″, which is the pole ſtar's diſtance from the pole for the middle of the year 1780.

250. The degrees in the circumference anſwer each to four minutes of time, and a quarter of one of theſe degrees anſwers to a minute of time; and the parallel upright ſtraight lines drawn from the points in the circumference to the numbered horizontal diameter, ſhew amongſt the concentric circles horizontally how many minutes the correſpondent point in the circumference is above or below the pole; and upward or downward, amongſt thoſe circles, how much it is eaſtward or weſtward from the pole.

251. From which conſtruction, knowing the day of the month, and hour and minute of the day, it may be readily known by inſpection, how many minutes the pole ſtar is elevated above, or depreſſed below, the pole at that time, and how many minutes it is eaſtward or weſtward of the pole.

Examples.

252. Examp. 1. What is the pole ſtar's ſituation April 2, at noon, for 1780? Look for April 2, and 0 hours under it, the height above the pole is ſhewn by the concentric circles 112 minutes, and the eaſting or weſting 0 minutes.

Examp. 2. What is the pole ſtar's ſituation April 2, at ſix in the evening? Look for April 2, and 6 hours, to the left, or weſtward thereof, is the place of the pole ſtar, of equal height with the pole, but 112 minutes diſtance weſt thereof.

Examp. 3. What is the ſituation of the pole ſtar April 2, at four in the morning? This is 8 hours ſhort of noon, or 16 hours of the preceding day; therefore count 8 hours to the right or eaſtward, or which is the ſame thing, 16 hours to the left or weſtward, and from that point, croſſing the horizontal diameter upwards, gives a place in the circumference 55′ 30″ higher than the pole, and 97′ eaſt of the pole.

253. In more general terms, the pole ſtar is north above the pole, or nearly ſo,

May 3 at 22^h or 10 in the morning.
June 3 at 20^h or 8 in the morning.
July 2 at 18^h or 6 in the morning.

Auguſt

Auguſt	1	at	16^h or 4 in the morning.
September	2	at	14^h or 2 in the morning.
October	5	at	12^h or midnight.
November	5	at	10 in the evening.
December	4	at	8 in the evening.
December	31	at	6 in the evening.
January	28	at	4 afternnon.
February	28	at	2 afternoon.

OBSERVATION.

254. From this conſtruction, the times of the pole ſtar's being at the greateſt height above the pole, the leaſt height below it; and at the greateſt elongation eaſt and weſt thereof, will eaſily be known, and all other ſituations whatſoever. For, if you find nearly the day of the month by the outermoſt circle, the hours under it ſhew the time of the pole ſtar's being above the pole on that day; and if the time of the day be paſt noon, count from that point as many hours to the left as the hours paſt noon; but if it be before noon, count as many hours as it is ſhort of noon to the right of that point; and in either caſe you come to the place of the pole ſtar at that time; from which, by help of the parallel ſtraight lines and concentric circles, its height above the pole, or depreſſion below it, and its elongation eaſtward or weſtward from the pole, will appear by inſpection.

OBSERVATION.

255. The converſe of the problem is manifeſt; for having found the time of the pole ſtar's being over the pole on the given day, count how many hours and parts of an hour that point is, either from its greateſt eaſtern or weſtern elongation, or being below the pole; but contrarily to the former way, and it is the time required.

256. As this method is wholly new, ſo the utility of it is very ample, particularly in ſettling the latitudes of places to the greateſt accuracy, where the niceſt inſtruments are applied; and likewiſe in drawing a meridian line, or ſetting up marks therein, where the beſt inſtruments applicable for that purpoſe are uſed; a method, whereby either of theſe problems may be readily performed to the greateſt exactneſs, whenever the pole ſtar can be ſeen through the teleſcope of an inſtrument, and which aſtronomers have never yet practiſed;

practiſed; but, inſtead thereof, confined their obſervations for the latitude to the ſituation of the pole ſtar above and below the pole, and for the meridian marks, either to theſe or the ſtar's greateſt eaſtern and weſtern elongation. I have made this method more perfect, by means of a proper delineation, whereby having the right aſcenſion of the ſun and of the pole ſtar, theſe particulars may be known to any exactneſs that can be requiſite.

I have been the more particular in deriving the obliquity of the ecliptic, and the annual approach of the pole ſtar towards the pole, from original obſervations made at the diſtance of long periods of time, rather than making uſe of the numbers which are given by the modern aſtronomers, becauſe they differ a little from one another; and to make this work the more an original work, being perſuaded that none of the modern aſtronomers can have recourſe for the ſettling of theſe things to better materials. The longitude of the pole ſtar may be now conſidered as being between the 25th and 26th degree of Gemini; and therefore, on account of the receſſion of the equinoctial points, or rather the apparent preceſſion of the ſtars in longitude, the pole ſtar will be approaching nearer towards the pole for near 330 years to come, after which time it will begin to recede from the pole. It appears from the obſervations, that this ſtar has not approached towards the pole more than 19,44″ per year at a mean for 150 years paſt; and from the problem itſelf, it muſt be now approaching rather ſlower than it has been during that time. Notwithſtanding this, M. de la Caille gives the annual approach now 19,69″, and the Connoiſſance des Temps 20″ per year; both of which are greater than the preceding annual approach towards the pole.

Experiment.

257. Having ſet a clock nearly to mean ſolar time by the method before deſcribed, and found the time when the pole ſtar would be on the meridian above the pole; I put up two lines in the plane of the meridian after the manner of the former experiment; and by theſe I obſerved the tranſits of circumpolar ſtars, and the tranſit of the pole ſtar over the meridian both above and below the pole.

258. The like experiment was made when the pole ſtar was below the pole.

Experi-

EXPERIMENT.

259. In like manner the lines were put up when the pole ſtar was at its greateſt eaſtern elongation; likewiſe at the greateſt weſtern elongation, and to know how many degrees and minutes of a degree the lines were then out of the plane of the meridian, we have the following proportion. As the coſine of the latitude : to radius : : ſine of half the pole ſtar's diſtance from the pole : to the ſine of half the horizontal diſtance of the pole ſtar from the pole, when it is at its greateſt elongation. From which proportion the ſolution may be had by the parallactic triangle as follows.

260. Lay a ſtraight ſlip of paper from the centre C of the triangle to the latitude of the place in the arch B D &c. and guiding your eye from 3360 in the line C B, upward until you come to the edge of the paper, from thence guide your eye to the right in the direction of the concentic circles, till you return to the line C B, and the graduated diviſions, reckoning from C, ſhew the half number of ſeconds, which the pole ſtar at its greateſt elongation is from under the pole. This doubled and divided by 60, gives the degrees and minutes from which the meridian line is to be ſet off by the large ſemi-circle as before deſcribed.

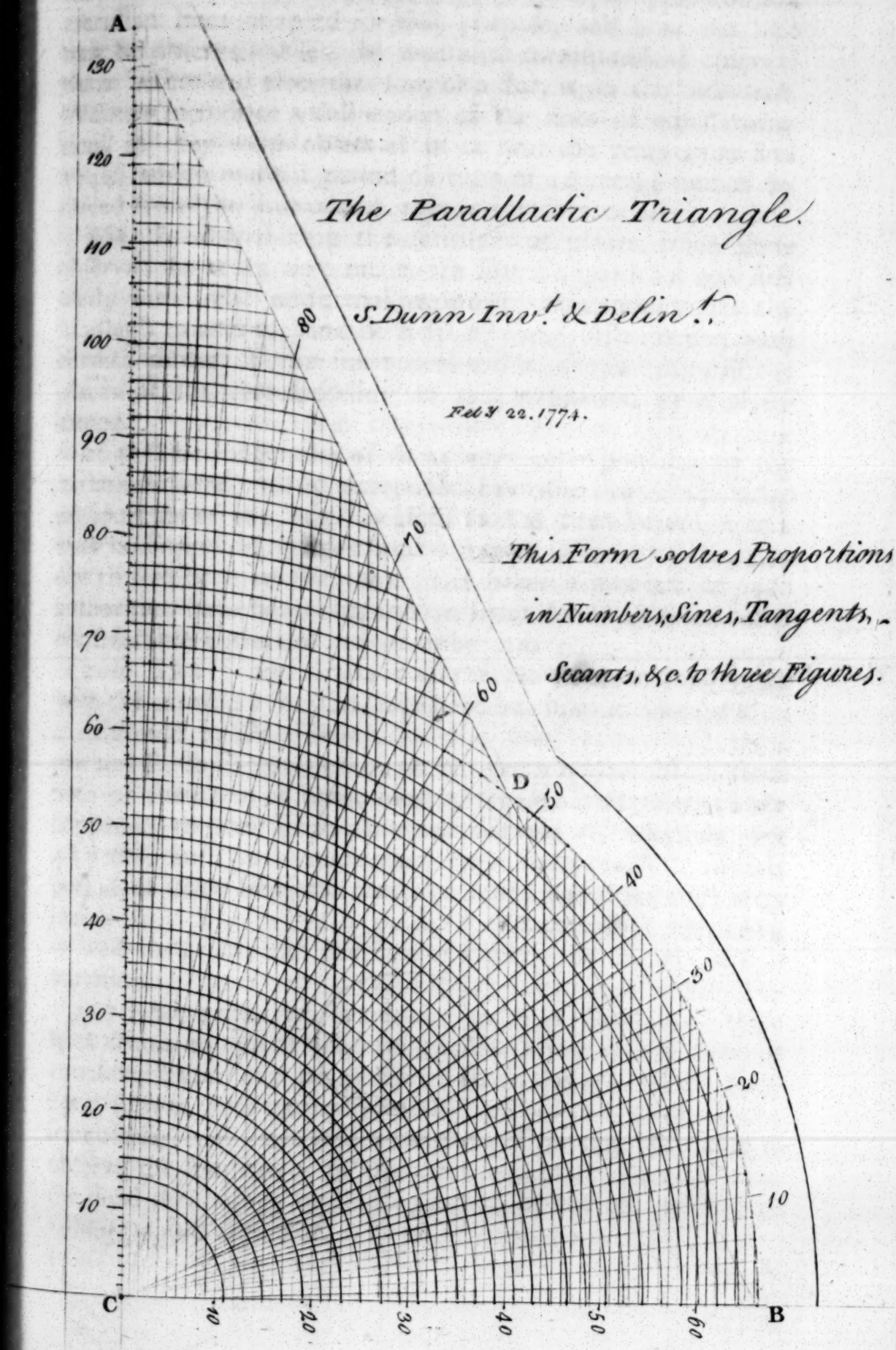
The ſame might have been effected by the polar delineation itſelf.

261. Having put up the meridian lines by the pole ſtar, the variation of the magnetic needle may be taken by it as before deſcribed, when the line was put up by obſervations of the ſun. And the change or alteration of that variation from time to time, but particularly from one year to another, may hereby be obſerved.

262. Near the equinoctial in north latitude the pole ſtar will not be applicable, on account of its nearneſs to the horizon. In ſuch places, on account of the ſun's great meridian height, it may be proper to continue the line quite over the hooks 1 and 2, before deſcribed in plate of lines, and in a place where the ſun may ſhine upon the line; and then the manner of making obſervations and drawing concluſions, will be exactly the ſame as though they were put up perpendicular to the horizon.

263. Having ſhewn the method whereby the horizontal deflection of the magnetic needle from the true meridian

may



The Parallactic Triangle.

S. Dunn Inv.t & Delin.t.

Feb.y 22. 1774.

This Form solves Proportions in Numbers, Sines, Tangents, Secants, &c. to three Figures.

may be taken on land by the help of a ſimple apparatus and meridian lines adapted for that purpoſe, and how the like may be obſerved at ſea, by means of correſpondent obſervations before and after the ſun, or a ſtar, is on the meridian. We now introduce a delineation of the lines of equal variation, as they were obſerved in or near the years 1700 and 1744, which made a period of time of 44 years; and as deduced from the journals of voyagers and travellers.

264. In determining the latitudes of places, when theſe obſervations at ſea were made, the Davis's quadrant was firſt uſed, with ſome additional improvements; afterwards the Hadley's quadrant; and by help of theſe inſtruments, with correct tables of the ſun's declination, the latitudes of the places at ſea, correſponding to the variations, were aſcertained.

265. The longitudes of the places correſponding to the variations were chiefly aſcertained from the dead reckonings or journals of the ſhips; which, as has been before noted, may be ſuppoſed to have been defective in ſome caſes, from one to two or more degrees. But when a number of ſuch authorities have been conſulted, it may be ſuppoſed that a conſiderable correction was thereby made.

266. The fainter lines, marked A, are nearly agreeing with the numbers for the reſpective latitudes and longitudes, as exhibited by Dr. Halley, for the year 1700, and thoſe marked B, for 1744 nearly, or 44 years after. And from both of theſe ſets of lines, may be ſeen what alterations have happened to the lines of equal degrees of variation for 44 years, and how much they have been removed in that period of time from the latitudes and longitudes they were before in. The letter *e*, prefixed, denotes eaſt variation, *w* weſt variation, and the figures expreſs the degrees of that variation.

267. Although the ſerpentine form of ſeveral of theſe lines ſeems to prove that no regular order of the lines of equal variation can be expected from the laws of the magnetic ſyſtem, it is naturally to be expected that part of this irregularity may have ariſen for want of having been able to obſerve the longitudes of places at ſea, more correctly than the dead reckoning, or ſhip's journal, by the old method of ſailing, would admit of.

268. It appears by this delineation, which is of the globular kind, and thereby better adapted for ſhewing the regularity or irregularity of the lines, that, in the Ethiopic ocean, ſeveral of the lines of equal variation have almoſt as much regularity as the meridians themſelves, in reſpect to their curvature; and that the variation or change that has been produced amongſt them, for near half a century paſt, has been as it were by removal of the curves of equal variation more weſtwardly, and at the ſame time ſomewhat altering the flexure.

269. From conſiderations ſomewhat like theſe it was, that, ſeventeen years ago, I propoſed a method, in my book of the Planiſpheres, of finding the longitude at ſea, by having the latitude of the place, and the variation of the compaſs, as delineated on a map or chart. But as I do not ſuppoſe that any delineations of the variation on maps or charts have been hitherto made accurate enough for the purpoſe of finding the longitude far off from the ſea ſhores, by the variation and the latitude; nor that both the longitudes of places near the ſea ſhores, and the variations at thoſe places, have been hitherto aſcertained with that accuracy which is requiſite; it is the peculiar deſign of this work, to ſhew as accurately as the nature of the problem will admit, how both of theſe things may be effected.

270. Although the delineation annexed be drawn to a ſmall ſcale, let it be applied to the finding of the longitude by this method, and the reſults compared with obſerved longitudes, as by the following examples:

271. At the Cape of Good Hope, the ſouthernmoſt promontory of Africa, the variation of the magnetic needle is ſaid to have been as follows:

		°	′	
Near the years	1600	0	0	
and	1622	2	0	weſt.
and	1675	8	0	weſt.
and	1700	10	0	weſt.
and	1722	14	30	weſt.
and	1744	16	30	weſt.
and	1756	18	0	weſt.
and	1766	19	40	weſt.

From

From which numbers it appears, that the yearly change of the variation at the Cape, from 1600 to 1766, has been as follows:

From 1600 to 1620	$5\frac{1}{2}'$	per year.
From 1620 to 1675	$6\frac{3}{4}$	
From 1675 to 1700	5	
From 1700 to 1722	12	
From 1722 to 1744	$5\frac{1}{2}$	
From 1744 to 1756	$7\frac{1}{2}$	
From 1756 to 1766	9	

Other accounts, which I have seen, make it more than 20° at the Cape for 1766.

272. It is probable that such irregularities may have arisen partly from erroneous observations, and partly from the different places where the observations were made. For, if we take a medium of the whole increase of variation, from the year 1600 to the year 1766, it is near 20 degrees, or $7\frac{1}{4}'$, per year. From the year 1684 to 1702, the mariners used to reckon it increasing at the rate of $9\frac{1}{2}'$ per year. And therefore it is easy to predict the variation for one, two, or three years to come, to some tolerable degree of accuracy, when it is known for certain to be either increasing or decreasing, and the mean yearly increase or decrease is nearly ascertained.

273. The observations made in and near London, for 190 years past, seem to indicate inequalities no less inconsistent with any regular theory. For the variation is said to have been observed by

		°	′	
Mr. Burrows	1580	11	17	east.
Mr. Gunter	1622	6	15	east.
Mr. Gellebrand	1634	4	4	east.
Mr. Bond	1657	0	0	
Mr. Seller	1666	0	34	west.
Sir Nicholas Millet	1670	2	6	west.
Dr. Halley	1672	2	30	west.
Dr. Halley	1692	6	0	west.
Dr. Halley	1700	8	0	west.
Mr. Graham	1722	14	15	west.
Per Chart	1744	17	15	west.
Mr. Canton	1759	19	8	west.
———	1772	20	55	west.

From 1580 to 1772 is 192 years; and the whole change of variation in that time has amounted to 32°, which, at a medium, has been nearly 10′ per year. Dr. Halley says, in 1701, the variation was $7°\frac{1}{2}$ in all parts of the channel, and that it increased a degree in $5\frac{1}{2}$ years, which is near 11′ per year.

From these observations it appears, that the yearly change of the variation of the magnetic needle, at London, has been nearly as follows:

From	1580	to	1622	7 decreasing.
From	1622	to	1634	11 decreasing.
From	1634	to	1657	10 decreasing.
From	1657	to	1666	14 increasing.
From	1657	to	1672	8 increasing.
From	1672	to	1692	11 increasing.
From	1692	to	1700	11 increasing.
From	1700	to	1722	17 increasing.
From	1722	to	1744	8 increasing.
From	1744	to	1759	8 increasing.
From	1759	to	1772	8 increasing.

274. Such irregularities as these seem to be hardly reconcileable with any theory of the magnetic system, and therefore the variation the more difficult to be predicted for any place of the earth or sea. Nevertheless, if the variation be observed at the end of shorter periods of time, the intermediate ones may be determined, and, for a few years, continued without much error. But, as was said before, part of those irregularities may have arisen from erroneous observations, and part from imperfect instruments. The boxes, in which magnetic needles are placed, are frequently made of brass: a metal, which, for more than twenty years ago, I was informed, by a most ingenious compass-maker, has frequently so much of the lapis in its composition, as to divert the needle a degree from its proper direction; I could have said more, and that may account for some of the inequalities here mentioned. The situation of these observers was likewise very different; Burrows observed at Limehouse; Gunter there; Gellebrand, at Deptford; Seller, in Wapping; Sir Nicholas Millet, at Battersea; and some in London; and it may be supposed, that their observations were

were partly affected by that daily oſcillation which happens to the needle, and which makes it requiſite to take ſeveral obſervations to come to the greater certainty.

275. The obſervations made at Paris, for more than 190 years paſt, ſeem to indicate great irregularity in the poſition of the magnetic needle. We find them as follows:

		°	′	
In the year	1580	11	30	eaſt.
——————— in	1610	8	0	eaſt.
——————— in	1640	3	0	eaſt.
——————— in	1666	0	0	
By Dr. Halley there in	1681	2	30	weſt.
— De la Hire in	1686	4	30	weſt.
— the ſame in	1699	8	10	weſt.
— the ſame in	1700	8	12	weſt.
— the ſame in	1701	8	48	weſt.
— the ſame in	1706	9	48	weſt.
— the ſame in	1711	11	2	weſt.
— the ſame in	1716	11	45	weſt.
— the ſame in	1719	12	20	weſt.
— M. Maraldi in	1759	18	10	weſt.
— the ſame in	1760	18	20	weſt.
— the ſame in	1765	19	0	weſt.
— P. Cotte in	1772	19	55	weſt.

From theſe obſervations, the magnetic needle has altered its poſition at Paris 31° 25′ in 192 years, which, at a medium, is near $9'\frac{7}{8}$ per year. In like manner, if the yearly change of the variation at Paris be inferred from the obſervations, it will be

	′	
From 1580 to 1610	7	decreaſing.
From 1610 to 1640	10	decreaſing.
From 1640 to 1666	7	decreaſing.
From 1666 to 1681	10	increaſing.
From 1681 to 1686	24	increaſing.
From 1686 to 1699	16	increaſing.
From 1699 to 1701	19	increaſing.
From 1701 to 1711	$13\frac{1}{2}$	increaſing.
From 1711 to 1719	10	increaſing.
From 1719 to 1759	9	increaſing.
From 1759 to 1765	8	increaſing.
From 1765 to 1772	8	increaſing.

276. The

276. The change which the poſition of the needle undergoes, from one part of the year to another, but chiefly that from one part of the day to another, are circumſtances which do in ſome meaſure take off that certainty, which might otherwiſe attend obſervations made by it. Theſe irregularities in the change, from one time of the year to another, may be admitted to amount in ſome caſes to near half a degree, and the daily change to near a ſixth of a degree: ſo that, could the magnetic variation be ever ſo truly predicted, except what irregularity ariſes from theſe circumſtances, the application of the variation, for the determination of the longitude, would, at ſome particular times, be attended with a proportional uncertainty. But one part of the objection is removed, by conſidering the nearneſs of ſome of the lines of variation on the ſeas; and the other part of the objection is removed, by taking a medium of ſeveral obſervations on the land.

277. We find many places on the ocean, where the diſtances of places, and the lines of unequal variation, are nearly as 8 to 5; that is, in 8 degrees, or 480 miles diſtance, the variation alters 5 degrees; therefore, in 48 miles of diſtance, it may be ſuppoſed to alter half a degree of variation; and here an error of half a degree in the variation would produce an error of 48 miles in diſtance. But, on the contrary, there are other places, where but 5 degrees of diſtance produce a difference of 10 degrees in variation; in which latter caſe, an error of 30 miles in diſtance anſwers to a degree in variation; and 15 miles in diſtance anſwers to half a degree in variation. And, even at ſea, a medium of ſeveral obſervations may be inſtrumental for removing an error that might be introduced by a dependance on any ſingle obſervation, in either of theſe caſes. And it is known, that the medium of ſeveral obſervations being properly taken, and properly applied, might make the error much leſs.

278. In ſome other parts of the ocean, the difference of variation is not ſo great within ſuch ſmall diſtances; but here the errors would be, in a great meaſure, expunged by a more certain method of obſerving the variation. And it is probable that, in ſuch places, the diurnal and annual variations may be leſs conſiderable.

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AMERICA.

Meridian of Cape Good Hope 18° 23' East of Greenwich.

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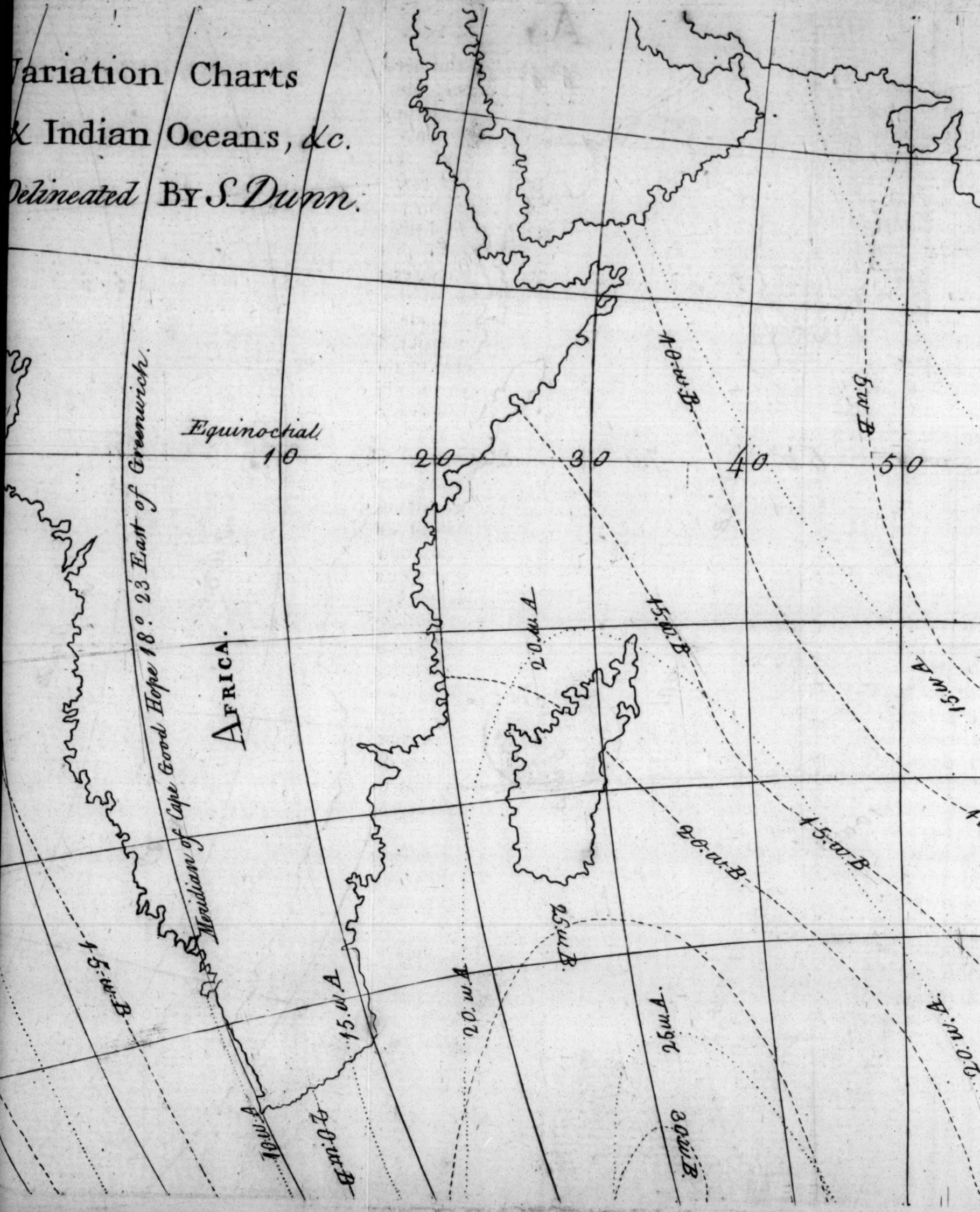
Variation Charts
& Indian Oceans, &c.
Delineated BY *S. Dunn.*



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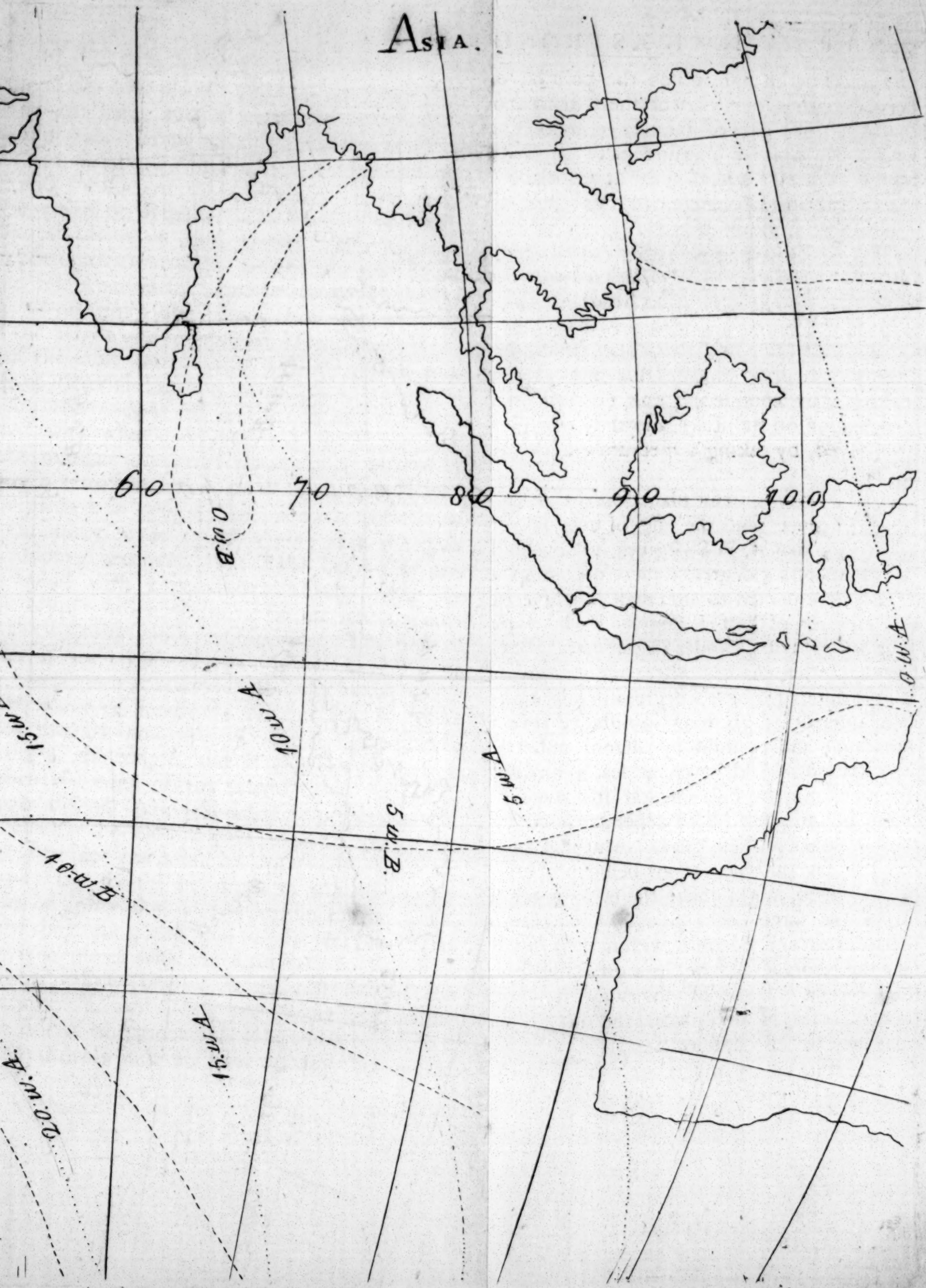
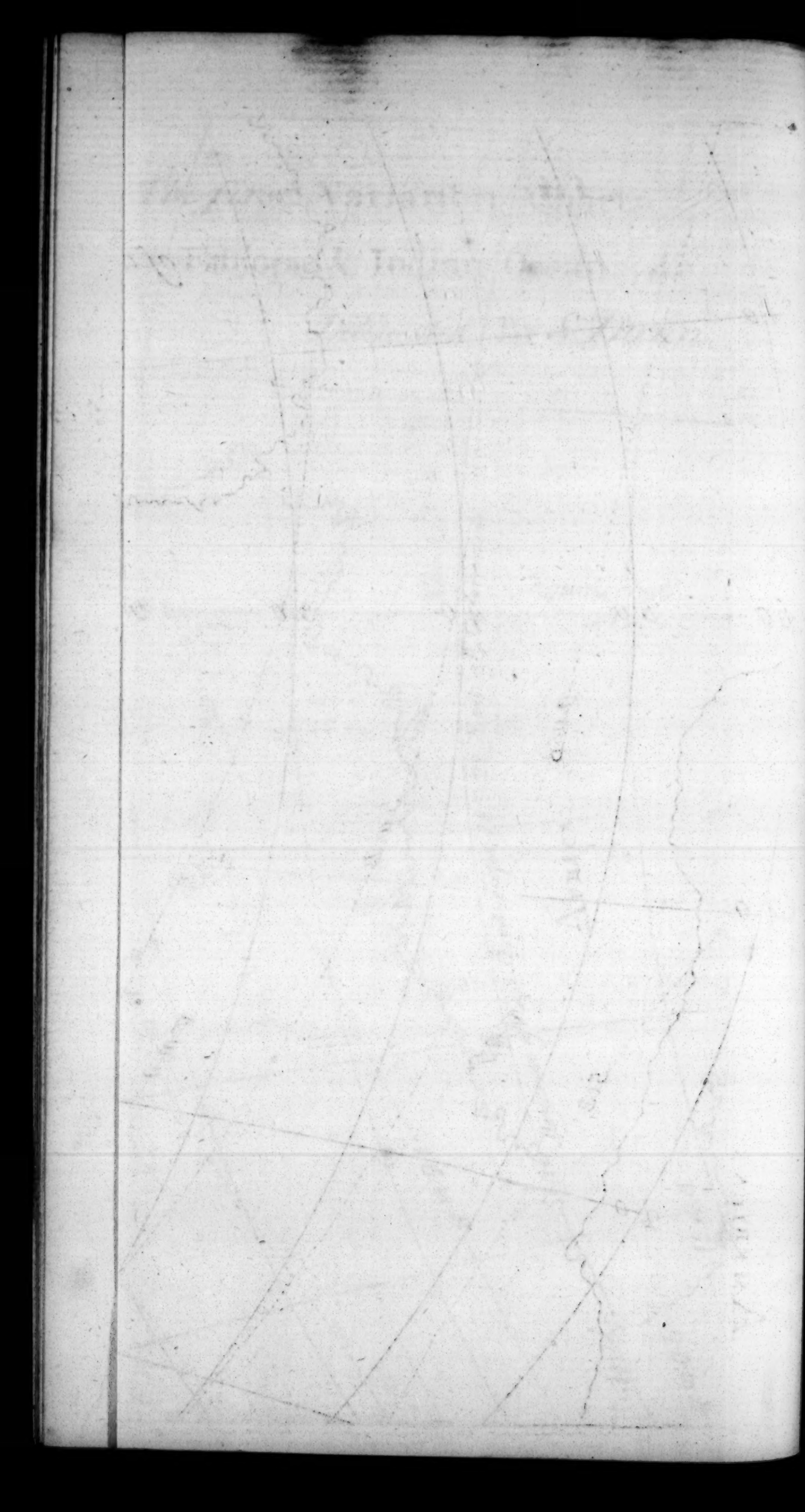




tudes and longitudes of places ſhould be correctly aſcertained, both on land, and at ſea, where the lines of equal variation do paſs. Many of theſe latitudes and longitudes are known ſufficiently exact; but the greater part are incorrect and inſufficient for application, in reſearches relative to the magnetic ſyſtem. The method of ſettling the longitudes of places on land is very eaſy, if proper times and opportunities are uſed; and the like may be ſaid of finding the longitude at ſea: but to determine the longitude at ſea, at all times and places, is a problem which has exerciſed the height of every human invention.

280. One of the moſt antient, and perhaps of the moſt eaſy, methods of determining the geographical longitudes of places, is, by obſervations of lunar eclipſes; and when it is conſidered with what eaſe theſe eclipſes can be obſerved, and the number of places whoſe longitudes have been accurately determined by them, it is ſurprizing they ſhould be ſo neglected in geographical affairs. Three objections are commonly made againſt them; firſt, that they happen but ſeldom; ſecondly, that their beginning and ending are affected with a penumbra; thirdly, that the middle of the eclipſe will not always be an inſtantaneous phænomenon to two correſpondent obſervers. The firſt of theſe objections is no argument againſt their utility, one or two good correſpondent obſervations being ſufficient for ſettling the difference of longitude of two places. The ſecond objection may be removed, by beginning to obſerve for the beginning of the eclipſe, and leaving for the end of the eclipſe, when the edge of the moon either begins or ceaſeth to appear diſtinct and well-defined to the eye of the obſerver, whether he uſeth a glaſs or not. As to the third objection, it is not intended, by obſervations of thoſe eclipſes, that any uſe ſhould be made of a medium between the times of immerſion and emerſion of the ſpots, nor between the beginning and end of the eclipſe. The third objection is removed with the ſecond.

281. The lunar eclipſe begins on the eaſt ſide of the moon, and ends on the weſt ſide; and before the eclipſe begins, the eaſt ſide of the moon will ſometimes be affected with an unuſual dulneſs half an hour of time before the eclipſe begins.

282. The

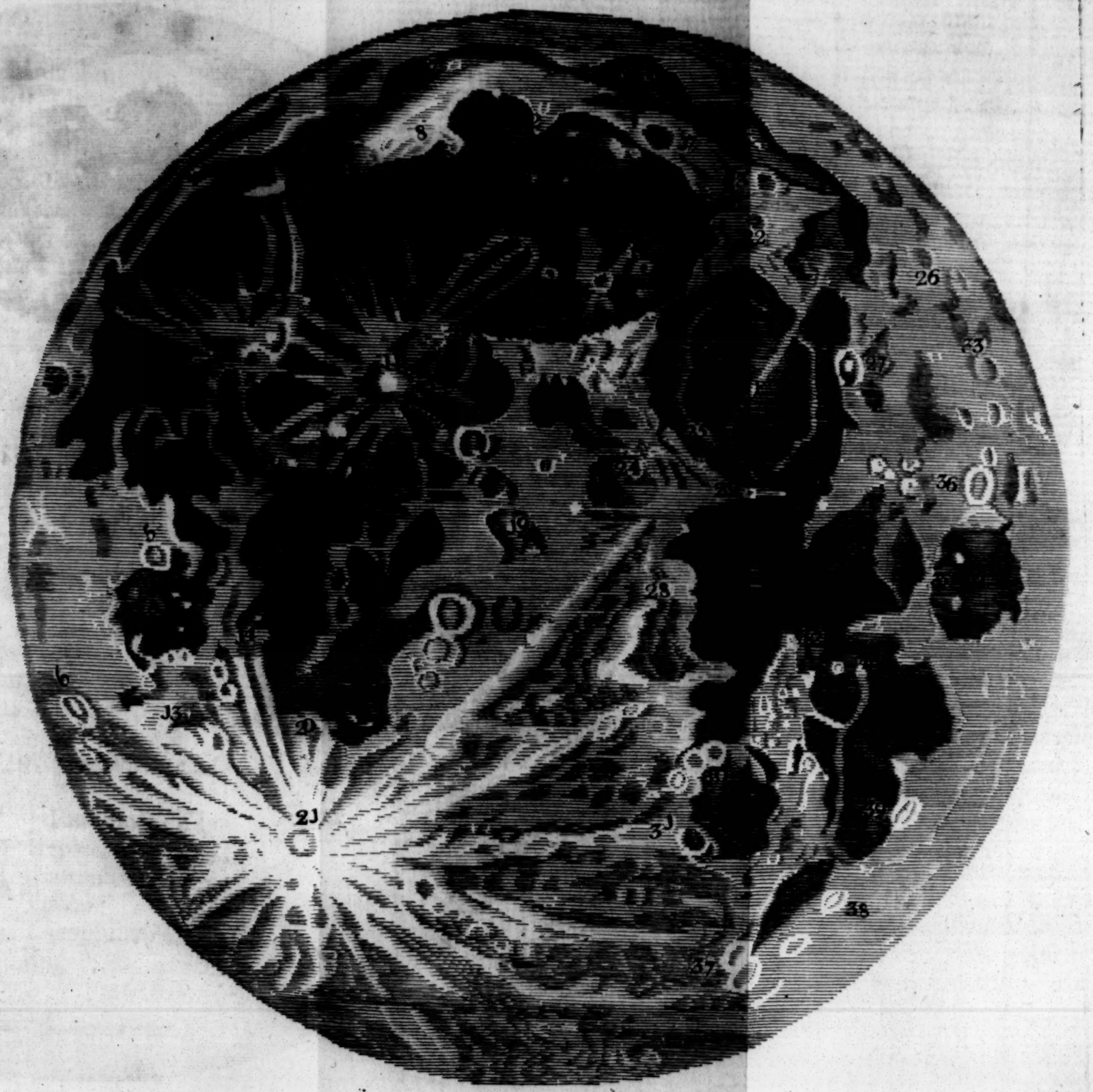
282. The clock, or time-keeper, which the aſtronomer intends to uſe, ſhould be near him; and if he intends to obſerve only the beginning and ending, and the touch of the ſhadow to the larger brown ſpots on the moon, the buſineſs may be pretty well effected by the naked eye only; eſpecially when it is clear weather, and the moon happens to be near the horizon; for the moon will then appear advantageouſly through the horizontal vapours. But if he intends to obſerve the ſpots to greater accuracy, a teleſcope or glaſs magnifying from 5 to 20 times will not be unſerviceable.

283. The beſt of all lunar eclipſes, for the purpoſes of the longitude, are ſuch as are total, becauſe in ſuch eclipſes the ſhadow comes on progreſſively, almoſt at right angles to the moon's viſible path, and thereby makes the inſtant of the appulſe or contact to the ſpots the more certain; otherwiſe, as in partial eclipſes, when the line of the ſhade appears to move obliquely by the ſpots, the contact is not ſo inſtantaneous, and therefore the phænomenon the more uncertain.

284. The lunar ſpots may be conſidered as of two kinds, either ſuch as are ſmall, round and well defined, or ſuch as are broad, irregular, and take up a conſiderable ſpace on the moon's diſc or face. The former kind are beſt to be applied in making obſervations of the paſſage of the ſhadow by them, in order thereby to determine the longitudes of places; notwithſtanding this the latter may be profitably applied, when obſervations are made of the ſhadow touching or leaving them, or centrally biſecting them, if they are not too large.

285. The lunar ſpots have the names given to them of many eminent aſtronomers, and in obſerving the contact of the ſhadow of the earth to any one of the ſmall, round or well defined ſpots, three things ſhould be attended to, firſt, the inſtant when the ſpot is touched; ſecondly, when it is centrally biſected; thirdly, when it is quite covered; theſe three, as the eclipſe comes on; and from the medium of them, the time of the central biſection may be inferred to a great exactneſs. The converſe is to be obſerved at the going off of the eclipſe. When the eclipſe is obſerved with a teleſcope magnifying 20 times, the central biſection of the ſpot may be taken by this method to 5 ſeconds of times,

A Map of the Moon.



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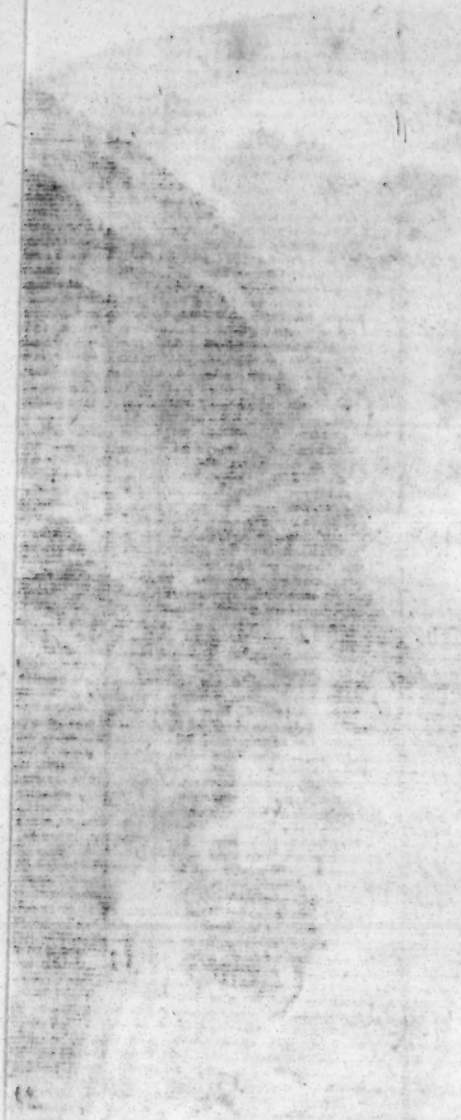
A



time when care, ingenuity, and a good clock are not wanting. And a fourth part of that accuracy would contribute greatly to geographical improvements. But many of theſe ſpots may be obſerved at the time of the ſame eclipſe, and the medium from theſe would leſſen the errors.

286. Such eclipſes can be obſerved at ſea; for glaſſes may be uſed there, whoſe field of view are large, when they do not magnify much, and ſuch are proper for the purpoſe; but the greateſt care and attention ſhould be employed in getting the clock or watch, by which the obſervation is made, either to mean or ſolar time, or in determining how much it is either too ſoon or too late.

287. The manner of regiſtering obſervations of a lunar eclipſe is thus:

1757, July 30, watch gains (or loſes) ′ per day.

h	′	Immerſions.	h	′	Emerſions.
10	1	Beginning.	11	56	Plato.
10	10	Grimaldus.	11	59	Ariſtarchus.
10	11	Gaſſendus.	12	0	Ariſtotle.
10	25½	Tycho.	12	6	Grimaldus.
10	32½	Lanſberg.	12	11	Kepler.
10	34	Kepler.	12	13½	Poſſidonius.
10	38	Copernicus.	12	27	Skikardus.
10	44	Manilius.	12	33	Manilius.
11	11	Plato.	12	36	Manelaus.
11	15	Poſſidonius.	12	39½	Tycho.
			13	4	Langrenus.
			13	7	End.

288. 1764, March 17, at 10^h 3′ 48″ Regulus tranſits the meridian.

h	′	Immerſions.	h	′	Emerſions.
10	39½	Beginning.	12	17½	Kepler.
10	40½	Skikardus.	12	20½	Copernicus.
10	51	Tycho.	12	57	Tycho.
10	53	Grimaldus.	13	6	Theophilus.
11	7	Galileus.	13	2	End.
11	12	Kepler.			
11	18	Copernicus.			
11	41	Manilius.			

This eclipſe was obſerved at Berlin, by M. Reccard; thus,

Beginning uncertain.				Tycho immerges.			
h	′	″		h	′	″	
11	31	38	Berlin.	11	46	2	Berlin.
10	39	30	Chelſea.	10	51	10	Chelſea.
	52	8	difference.		54	52	difference.

Grimaldus immerges.				Kepler immerges.			
h	′	″		h	′	″	
11	47	17	Berlin.	12	6	6	Berlin.
10	53	0	Chelſea.	11	11	45	Chelſea.
	54	17	difference.		54	21	difference.

Tycho emerges.				End uncertain.			
h	′	″		h	′	″	
13	52	32	Berlin.	14	18	16	Berlin.
12	57	20	Chelſea.	13	22	10	Chelſea.
	55	12	difference.		56	6	difference.

Chelſea à Berlin.

	′	″	
By the beginning	52	8	uncertain.
By the end	56	6	uncertain.
By Tycho immerged	54	52	
By Grimaldus immerged	54	17	
By Tycho emerged	55	12	
By Kepler immerged	54	21	
By the medium	54	30	
True diff. Longitude	54	20	
Error		10	

The ſame eclipſe was obſerved at other places; but with no particular nicety, the beginning, middle, and end of the immerſions and emerſions of the ſpots.

Theſe two obſervations of lunar eclipſes were made with a common watch, an inſtrument not always to be depended on for a few hours together, eſpecially when it is taken out of the pocket, and expoſed to the cold air.

289. As

289. As there are ſo many immerſions and emerſions that may be obſerved, there is no neceſſity for being overnice as to the exact inſtant of any one of them; becauſe a compariſon of ſeveral correſpondent ones will be ſufficient for the purpoſe, when the obſervations are made to a quarter of a minute of time.

290. Suppoſing that but one ſuch eclipſe could be obſerved in a year, and this was thought to be but a ſlow proceeding, it would be a ſure one; for the longitudes of the places at ſea, where ſuch obſervations could be made, would thereby be determined as accurately as would be required, by a compariſon with the longitudes of other places, where the obſervations had been made on land. And ſuch obſervations at ſea may be made by many ſhips on the ocean, at great diſtances from each other, and at the ſame time, the longitudes of all thoſe places will be had with equal certainty, and their latitudes may in many caſes be nearly enough had by the dead reckoning, but more correctly from meridian altitudes, and the ſhip's difference of latitude before or after, compared with them.

291. As near to the time of ſuch an obſervation of a lunar eclipſe as poſſible, if it be obſerved at ſea, the variation of the magnetic needle ſhould be taken; and be continued to be taken, for the ſpace of a day, at ſeveral different intervals of time from each other; and by the medium of theſe variations the true variation ſhould be inferred. The problems and ſingle obſervations more immediately applicable in the doing of this are farther on in this work. Thus the variation of the needle ſhould be taken at as many places of the ocean as poſſible, and as the latitudes and longitudes of thoſe places will be nearly known from the obſervations of the eclipſe, thoſe places will be as ſtations, whereby the lines of variation may be drawn from the lands and ſea ſhores, and without conſiderable error extended to either great or ſmall diſtances, in the open ſeas and oceans.

292. The longitudes of places on land and near the ſea ſhores may be known by another method, correſpondent obſervations of the moon's tranſit over the meridians of the places of obſervation. As the moon comes to the meridian of a place frequently more than three quarters of an hour later on one day than on the preceding one, this

difference of 45′, or whether it be more or leſs, when it is nearly known, may be conſidered as proportional nearly to the whole 360° of longitude, or circumference of the equinoctial line. Therefore if any two correſpondent obſervers, at different places, and great diſtances from each other, have ſet their clocks alike either to ſiderial or mean or ſolar time, and they obſerve the moon on the meridian at the ſame hour, minute, and ſecond, by the clocks, the places are under the ſame meridian, otherwiſe not; and a proportional part will ſhew the difference of longitude nearly. This is one of the moſt critical obſervations that can be made for determining the longitudes of places on land, even by the application of the beſt inſtruments, for an error of 8″ of time between the time obſerved of the ſtar and moon's meridional tranſits may produce an error of a degree of longitude, and an error of 4″ of time an error of half a degree of longitude. But when many ſuch obſervations are made, though ſome of them ſhould err more, others may err leſs; and the medium would reduce thoſe errors, and make them inconſiderable.

293. In this method, the moon's ſemidiameter is variable, but ſuch increaſe or decreaſe cannot affect the obſervation unleſs the difference of longitude of the places is great; and even then it may be provided againſt by either proper obſervations or tables. And as great an error may ariſe in not allowing for the moon's increaſe in diameter, by a great meridian altitude, but the method of taking off an error on this account is ſhewn farther on. Allowing for both of theſe, the obſervation is to be made as follows.

OBSERVATION.

294.

	h	′	″	
				1771, December 10.
At	3	31	8	moon's tranſit obſerved at A.
	3	31	18	——— ——— at B.
			10	difference.

45′ moon's receſſion in 24 hours.

45′ : 360° :: 10″ : 1° 20′ difference longitude.

OBSER-

The Names of the LUNAR SPOTS.

1	Grimaldus	25	Menelaus
2	Galileus	26	Hermes
3	Aristarchus	27	Possidonius
4	Kepler	28	Dionysius
5	Gassendus	29	Pliny
6	Skikardus	30	Theophilus
7	Harpalus	31	Fracastorius
8	Hericlides	32	Promont.^{m} acutum
9	Lansbergius	33	Messalia
10	Reinoldus	34	Promont.^{m} somnii
11	Copernicus	35	Proclus
12	Helicon	36	Cleomedes
13	Capuanus	37	Snellius
14	Bulialdus	38	Petavius
15	Eratosthenes	39	Langrenus
16	Timocharis	40	Furuntius
17	Plato	A	Mare humorum
18	Archimedes	B	Mare nubium
19	Ins.^{a} sinus medii	C	Mare imbrium
20	Pitatus	D	Mare nectaris
21	Tycho	E	Mare tranquillitatis
22	Eudoxus	F	Mare serenitatis
23	Aristotle	G	Mare fecunditatis
24	Manilius	H	Mare crisium

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OBSERVATION.

295.

h ′ ″ December 11.
At 4 16 2 moon's tranſit obſerved at A.
4 16 13 ——————————— at B.
11 difference.
43′ moon's receſſion in 24 hours.
43′ : 360° : : 11″ : 1° 32′ difference longitude.

OBSERVATION.

296.

h ′ ″ December 12.
At 4 59 10 moon's tranſit obſerved at A.
4 59 25 ——————————— at B.
15 difference.
41′ moon's receſſion in 24 hours.
41′ : 360 : : 15″ : 2° 11′ difference longitude.

OBSERVATION.

297.

h ′ ″ December 13.
At 5 39 52 moon's tranſit obſerved at A.
5 40 1 ——————————— at B.
9 difference.
41′ moon's receſſion in 24 hours.
41′ : 360° : : 9″ : 1° 19′ difference longitude.

OBSERVATION.

298.

h ′ ′ December 18.
At 9 20 11 moon's tranſit obſerved at A.
9 20 28 ——————————— at B.
17 difference.
52′ moon's receſſion in 24 hours.
52′ : 360° : : 17″ : 1° 59′ difference longitude.

OBSERVATION.

299.

h ′ ″ Decemher 20.
At 11 6 20 moon's tranſit obſerved at A.
11 6 28 ——————————— at B.
8 difference.

57° moon's receſſion in 24 hours.
57′ : 360° :: 8″ : 0° 50′ difference longitude.

OBSERVATION.

300.

	h	′	″	
				1772, January 9.
At	3	22	5	moon's tranſit obſerved at A.
	3	22	17	——————— at B.
			12	difference.

57′ moon's receſſion in 24 hours.
57′ : 360° :: 12″ : 1° 15″ difference longitude.

301. Hence the medium of the ſeveral obſervations being taken, they will be as follows:

		°	′	
By obſervation	10th December	1	20	difference longitude.
———————	11th ———	1	32	———————
———————	12th ———	2	11	———————
———————	13th ———	1	19	———————
———————	18th ———	1	59	———————
———————	20th ———	0	50	———————
———————	9th January	1	15	———————
	The medium	1	29	the difference of

longitude of the two places, which was required. And this is one way whereby the longitudes of places on land may be ſettled to almoſt any degree of exactneſs, by the application of a little care and diligence, was there no other method whatſoever. For it has been before ſhewn how the tranſits of the ſun, ſtars, and the moon's enlightened limb, may be taken acroſs the lines without an error of a ſecond of time; and the catalogue of ſtars exhibits the right aſcenſions of ſuch of the firſt and ſecond magnitude, as are near the equator, and moſt proper for the purpoſe; and therefore the difference of time between the tranſit of a ſtar and that of the moon will give the moon's viſible right aſcenſion, when ſhe is near the meridian. Nor is it ſtrictly requiſite, that the lines ſhould be abſolutely in the plane of the meridian of obſervation; becauſe, were the lines put half a minute of time out of the plane of the meridian, which it is hardly poſſible for them to be, under the inſtruction herein delivered, they would

would give the place by obſervation but 17″ of a degree nearly different from the truih; for $24^h : 13^\circ :: 30'' : 17'$ of a degree; and this in moſt caſes would ſcarce exceed an error of 8 or 9 minutes in longitude.

302. The longitudes of places on land may be found by a third method, called the Immerſions and Emerſions of the Satellites of Jupiter. To treat of this method fully, in this part of the work, would require a digreſſion, and the explanation of ſeveral things not eaſily to be diſcuſſed in a few words. Such are therefore here poſtponed.

303. In the making of theſe obſervations, the aſtronomer ſhould be provided with a teleſcope magnifying not leſs than 30 times, and ſhewing the object ſharp, clear, and well-defined. His clock ſhould be ſet or adjuſted to time as for the other obſervations. He ſhould know how to find the planet Jupiter in the heavens amongſt the fixed ſtars; and the poſition of the ſhadow of the planet at all times and ſeaſons; and he ſhould be able to exert his attention in proportion to the difficulties he may meet with, in making an obſervation, when the planet happens to be nearly in oppoſition to, or conjunction with, the ſun.

304. The obſervations of the ſatellites, applicable for the longitude, are of three kinds. The immerſions, or diſappearing of the ſatellites; the emerſions, or appearing of them; and their conjunctions, or apparent paſſing by one another. In order to know when the immerſions and emerſions happen, it will be requiſite to have their predictions from the ſatellite tables, ſomewhat near the truth, otherwiſe an obſerver may be put to the diſagreeable neceſſity of attending the obſervation a long time, and after all not ſucceed, through the interpoſition of clouds or unfavourable weather. But ſuch predictions are not requiſite in obſerving the conjunctions.

305. When the immerſions, emerſions, or conjunctions are obſerved, the correſpondent obſervations being compared with each other, give the difference of longitude. Thus,

Observation.

306. 1761, July 22, an immerſion of the firſt ſatellite of Jupiter obſerved at different places, as follows:

At

	h	′	″	
At	12	34	46	by me at Chelſea.
	12	35	13	by another in London.
			27	Chelſea à London.

307.

	h	′	″	ANOTHER.
At	12	34	46	by me at Chelſea.
	12	35	29	at Greenwich.
			43	Chelſea à Greenwich.

308.

	h	′	″	ANOTHER.
At	12	34	46	by me at Chelſea.
	12	45	30	by M. Clugni at Paris.
		10	44	Chelſea à Paris.

309.

	h	′	″	ANOTHER.
At	12	34	46	by me at Chelſea.
	13	40	50	at Vienna.
	1	6	4	Chelſea à Vienna.

310.

	h	′	′	ANOTHER.
At	12	34	46	by me at Chelſea.
	13	45	47	at Tyrnaw, Hungary.
	1	11	1	Chelſea à Tyrnaw.

This ſatellite was 43 ſeconds of time loſing its light.

OBSERVATION.

311. 1761, Auguſt 10, an immerſion of the ſecond ſatellite of Jupiter obſerved.

	h	′	″	
At	10	29	35	by me at Chelſea.
	11	43	26	at Cape Good Hope.
	1	13	51	Chelſea à Cape Good Hope.

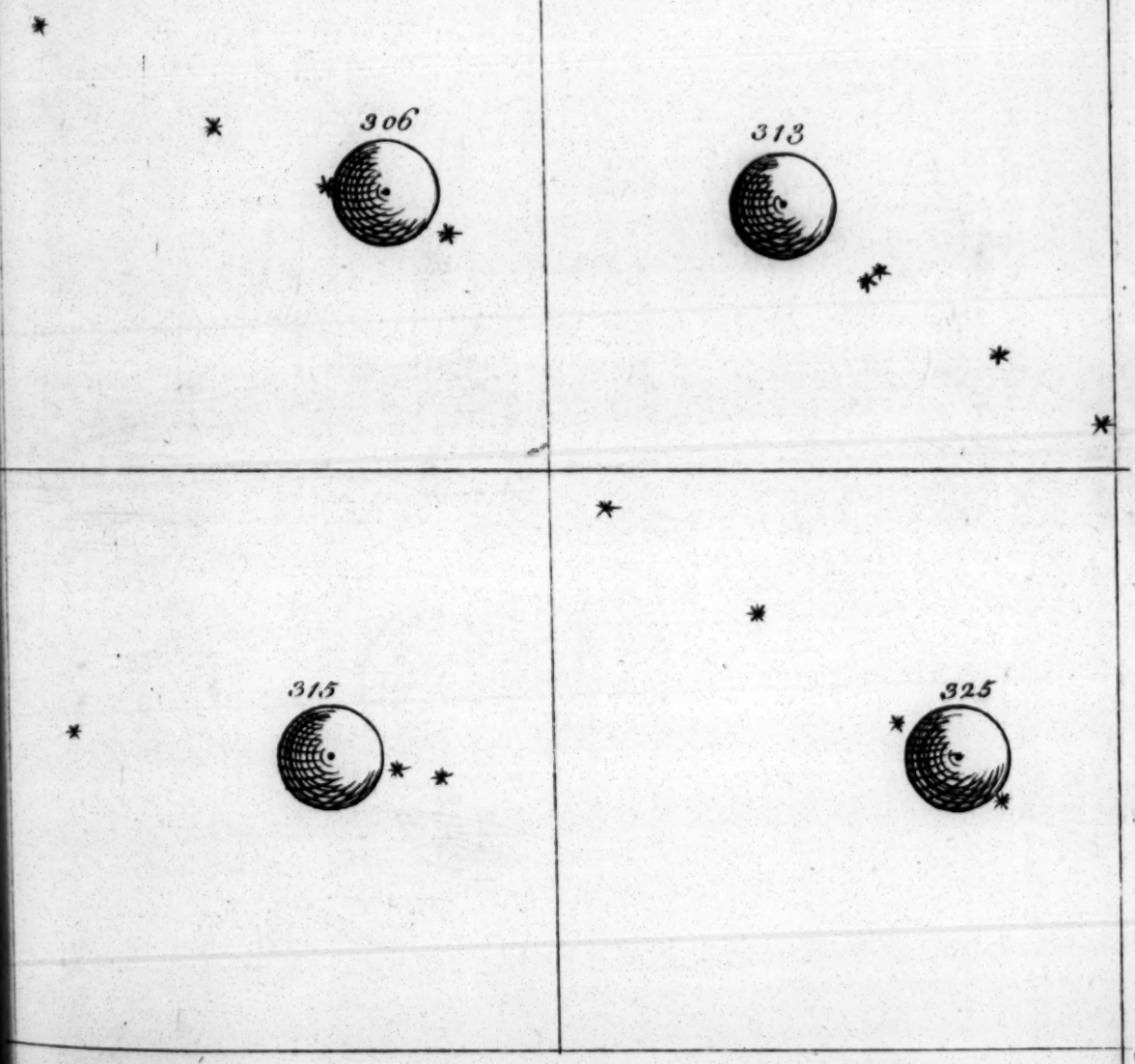
This ſatellite was 40 ſeconds of time loſing its light.

OBSERVATION.

312. 1761, Auguſt 10, an immerſion of the firſt ſatellite of Jupiter obſerved.

At

Observ'd Appearances of Jupiter & the ♃ Satellites.



From Observations the most instantaneous Conjunctions, are determinable to less than 20″ of Time, or 5 Miles Long.d at the Equator ♃.

S.D.

	h	′	″	
At	10	51	52	by me at Chelſea.
	12	5	46	at Cape Good Hope.
	1	13	54	Chelſea à Cape Good Hope.

This ſatellite was 50 ſeconds of time loſing its light.

OBSERVATION.

313. 1761, Auguſt 10, at $12^h\ 24'\ 21''$, a conjunction of two ſatellites obſerved by me at Chelſea; the inſtant of which was not uncertain to $30''$ of time. Had this been obſerved elſewhere, the longitude might have been inferred therefrom, with as much certainty as from the immerſions or emerſions.

OBSERVATION.

314. 1761, Auguſt 10, an immerſion of the fourth ſatellite of Jupiter obſerved.

	h	′	″	
At	14	18	12	by me at Chelſea.
	15	32	57	at Cape Good Hope.
	1	14	45	Chelſea à Cape Good Hope.

This ſatellite was 10′ of time loſing its luſtre, till it was wholly immerſed; a circumſtance, which, in different ſtates of the air, muſt make it unfit for the longitude.

OBSERVATION.

315. 1761, Auguſt 21, an immerſion of the firſt ſatellite of Jupiter obſerved.

	h	′	″	
At	14	41	19	by me at Chelſea.
	14	51	52	at Paris obſervatory.
	14	51	56	at Paris obſervatory.
	14	52	12	at Paris.
	14	52	18	at Paris.
	14	52	20	at Paris.
		10	37	Chelſea à Paris obſervatory.

From theſe obſervations made at Paris, it may be obſerved, that the beſt of practical aſtronomers may happen to differ almoſt half a minute of time in pronouncing the time of the immerſion of the firſt ſatellite of Jupiter, although they are at no great diſtance from each other. Theſe foreign obſervations may be ſeen in the Connoiſſance des Temps, 1767.

316. ANOTHER.

h	′	″	
At 14	41	19	by me at Chelſea.
15	56	2	at Cape Good Hope.
1	14	43	Chelſea à Cape Good Hope.

317. ANOTHER.

h	′	″	
At 14	41	19	by me at Chelſea.
15	48	19	at Vienna.
1	7	0	Chelſea à Vienna.

318. ANOTHER.

h	′	″	
At 14	41	19	by me at Chelſea.
15	52	45	at Tyrnaw, Hungary.
1	11	26	Chelſea à Tyrnaw.

At this obſervation, the ſky was perfectly clear with me; notwithſtanding which, the ſatellite was gradually diminiſhing in luſtre for more than a minute and an half of time. This circumſtance muſt neceſſarily have made the time of immerſion the more uncertain to perſons who obſerved through a groſs air, or in indifferent weather.

319. The ſame night, Auguſt 21, at 15° 48′ 0″, the ſecond ſatellite immerſed, after loſing its light in 41 ſeconds of time. No correſpondent obſervation.

OBSERVATION.

320. 1761, Auguſt 30, an immerſion of the firſt ſatellite of Jupiter obſerved.

h	′	″	
At 11	7	0	by me at Chelſea.
11	7	31	in London.
		31	Chelſea à London.

321. ANOTHER.

h	′	″	
At 11	7	0	by me at Chelſea.
12	13	21	at Vienna.
1	6	21	Chelſea à Vienna.

322. ANOTHER.

h	′	″	
At 11	7	0	by me at Chelſea.
12	18	9	at Tyrnaw, Hungary.
1	11	9	Chelſea à Tyrnaw.

323.

ANOTHER.

	h	′	″	
At	11	7	0	by me at Chelſea.
	12	21	32	at Cape Good Hope.
	1	14	32	Chelſea à Cape Good Hope.

324.

ANOTHER.

	h	′	″	
At	11	7	0	by me at Chelſea.
	12	58	50	at Cajanebourg, Sweden.
	1	51	50	Chelſea à Cajanebourg.

This ſatellite was 25 ſeconds of time loſing its light.

OBSERVATION.

325. 1761, September 8, an immerſion of the firſt ſatellite of Jupiter obſerved at $7^h\ 40'\ 5''$. The ſame evening, the ſecond ſatellite immerſed at

h	′	″	
10	28	40	by me at Chelſea.
11	42	20	at Cape Good Hope.
1	13	40	Chelſea à Cape Good Hope.

326. By theſe obſervations, the longitudes of places from Chelſea are as follow:

London. h ′ ″	Cape Good Hope. h ′ ″	Paris obſervatory. h ′ ″
0 0 27	1 13 51	0 10 33
0 0 31	1 13 54	0 10 37
	1 14 45	
0 0 29	1 14 43	0 10 35
	1 14 32	
Vienna.	1 13 40	Tyrnaw.
h ′ ″		h ′ ″
1 6 4	1 14 14	1 11 1
1 7 0		1 11 9
1 6 21	Greenwich.	
	h ′ ″	1 11 5
1 6 28	0 0 43	

327. And comparing the reſult of theſe obſervations with the moſt authentic ones that have been publiſhed;

	h	′	″
Chelsea à London at St. Paul's is	0	0	29
by others	0	0	15
Difference			14
	h	′	″
Chelsea à Greenwich by these observations	0	0	43
by others	0	0	37
Difference			6
	h	′	′
Chelsea à Paris by these observations	0	10	35
by others	0	9	55
Difference			40
	h	′	″
Chelsea à Vienna by these observations	1	6	28
by others	1	6	5
Difference			23
	h	′	′
Chelsea à Cape Good Hope by these	1	14	14
by others	1	14	9
Difference			5
	h	′	″
Chelsea à Tyrnaw by these observations	1	11	5
by others	1	10	13
Difference			52
	h	′	″
Chelsea à Cajanebourg by these	1	51	50
by others	1	51	30
Difference			20

And comparing all these differences together, they will be thus; for

	″	
Chelsea and St. Paul's, London	14	of time.
Chelsea and Greenwich	6	
Chelsea and Paris	40	
Chelsea and Vienna	23	
Chelsea and Cape Good Hope	5	
Chelsea and Tyrnaw	52	
Chelsea and Cajanebourg	20	
The medium	23	

328. I might

328. I might here inſert other obſervations of the ſatellites made by me, with their correſpondent ones made at other places, and their compariſons with one another; but ſhall only now obſerve, that theſe obſervations of mine were made with a very good reflector magnifying 55 times, and out of the groſſneſs which uſually attends the air of the metropolis. Theſe obſervations agree remarkably well with thoſe of Greenwich and the Cape; a little difference ariſeth being compared with London; nothing conſiderable with thoſe of Cajanenburg; but the difference ariſing between mine and thoſe at Paris, and Tyrnaw, may have ariſen partly from a difference of teleſcopes, or other cauſes.

We proceed to ſhew how the time is aſcertained by ſingle obſervations, with ſome other problems of the ſphere, uſeful in practical aſtronomy.

329. Aſtronomical calculations are many of them made by the principles of trigonometry; and trigonometry, or the doctrine of triangles, is of two kinds, plane and ſpherical. Plane trigonometry being the doctrine of triangles formed on a plane; and ſpherical, that of triangles formed on the ſurface of a ſphere.

330. Plane trigonometry is applicable to ſome parts of aſtronomy, where the triangles are very ſmall, becauſe a ſmall part of the ſurface of a ſphere or globe may, in many caſes, be conſidered as a plane; and this will frequently ſhorten the calculation, and make it very eaſy, without introducing any ſenſible error in caſes, which would be extremely difficult and tedious by the ſtrict principles of ſpherical triangles. This method is applicable in refraction, parallax, &c. when the differences which form one or more ſides of a triangle in the concave hemiſphere of the heavens are very ſmall; and therefore they may then be treated as right lines. But this method has its limits, and ſhould be introduced with caution.

331. Plane and ſpherical trigonometry have a great number of theorems, whereby aſtronomical obſervations may be ſo computed or calculated as to become of great utility in geography and navigation; but it is only the buſineſs of aſtronomers to apply them, when they are demonſtrable from geometrical or trigonometrical principles. Some of the moſt uſeful trigonometrical theorems are as follows:

THEOREM.

THEOREM.

332. Any ſide of a right angled plane triangle may repreſent radius, and the other ſides will reſpectively repreſent either a ſine, tangent, or ſecant; and there is the ſame direct proportion between the ſide repreſenting radius, and the radius; as there is between any one of the other ſides, and the ſine, tangent, or ſecant, which it repreſents.

THEOREM.

333. The ſides of all plane triangles, whether right angled or not, are in direct proportion to the ſines of their oppoſite angles.

THEOREM.

334. In oblique angled plane triangles, when two ſides and the included angle are given; As the ſum of the ſides, is to their difference; ſo is the co. tangent of half the contained angle, to a fourth term, which is the half difference of the unknown angles. This added to their half ſum, gives the greater; and ſubtracted from the half ſum, gives the leſſer angle ſought.

THEOREM.

335. In an oblique angled plane triangle, when a perpendicular is let down on the baſe, from the angle oppoſite thereto, dividing the baſe into two unequal ſegments or parts;

As the baſe :
Is to the ſum of the two other ſides : :
So is the difference of thoſe ſides :
To the difference of the ſegments of the baſe.

Half this difference added to the half baſe, gives the geater ſegment; and ſubtracted therefrom, gives the leſſer ſegment of the baſe.

THEOREM.

336. When the three ſides of a plane triangle are given to find an angle;

As the product made of the ſides which include the angle ſought :
Is to the product made by the half ſum of the three ſides, multiplied into the difference between that half ſum and the baſe : :

So

So is the ſquare of the radius :
To the ſquare of the co. ſine of half the angle ſought.

THEOREM.

337. Or, when the three ſides are given to find an angle;
As the product made of the ſides which include the angle ſought :
Is to the product made by, the half ſum of the three ſides leſſened by one of the ſides including the angle ſought; multiplied by the half ſum of the three ſides, leſſened by the other ſide including the angle ſought : :
So is the ſquare of the radius :
To the ſquare of the ſine of half the angle ſought.

THEOREM.

338. Or, when the three ſides are given, to find an angle;
As the product made by multiplying, half the ſum of the ſides; into half the ſum of the ſides leſſened by the baſe :
Is to the product made by, half the ſum of the three ſides, leſſened by one of the including ſides; multiplied into half the ſum of the three ſides, leſſened by the other including ſide : :
So is the ſquare of the radius :
To the ſquare of the tangent of half the angle ſought.

THEOREM.

339. Or, when the three ſides are given to find an angle;
As the product made by, half the ſum of the three ſides, leſſened by one of the including ſides; multiplied into half the ſum of the three ſides, leſſened by the other including ſide :
Is to the product made by, half the ſum of the three ſides; multiplied into half the ſum of the three ſides, leſſened by the baſe : :
So is the ſquare of the radius :
To the ſquare of the co. tangent of half the angle ſought.

THEOREM.

340. Or, when three ſides are given to find an angle;
As double the product made of the two ſhorter ſides :

Is

Is to the product made by multiplying, their ſum leſſened by the baſe; into the ſum of the three ſides : :

So is radius :

To the verſed ſine of the ſum of the angles at the baſe.

And,

As the baſe : to the ſum of the ſhorter ſides : : So is the co. ſine of the half ſum of the angles next to the baſe : to the co. ſine of half the difference of thoſe angles.

Which half difference added to the half ſum, gives the greater angle; and ſubtracted from the half ſum, leaves the leſſer angle.

341. The application of theſe theorems relative to plane triangles is very extenſive; in practical aſtronomy, they are of utility in determining the relative diſtances of the celeſtial bodies from the earth, and from one another; in determining their relative poſitions to one another; their apparent diameters in different parts of their orbits; the increments or decrements of their motions; and the effects of parallaxes, &c. But this is but a part of that trigonometry which is applicable to aſtronomy; nor is it poſſible to make any tolerable proficiency in this ſcience without ſpherical triangles, which are of another kind, and formed after a very different manner.

342. A ſpherical triangle is formed by three arches of a great circle of the ſphere, whether it be on the earth, or in the heavens. And a great circle is the largeſt that can be drawn on either the convex or concave ſurface of the earth or heavens reſpectively.

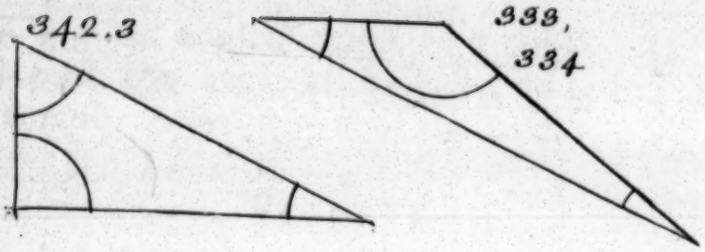
343. An angle of a ſpherical triangle is an arch of a great circle drawn from one of the containing ſides to the other, but it muſt be at the diſtance of 90° from the angular point; this requires the ſides containing the angle ſometimes to be lengthened, and at other times to be ſhortened.

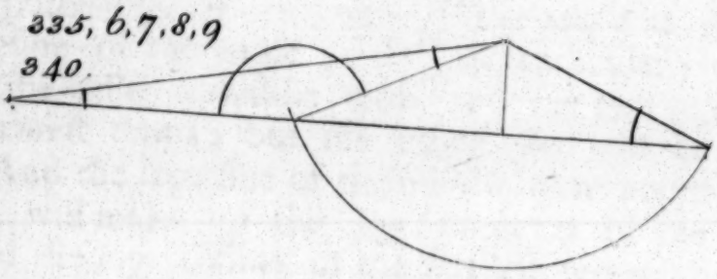
344. When a ſpherical triangle has one right angle, it is a right angled ſpherical triangle; when none of its ſides nor angles are 90°, it is commonly called an oblique angled ſpherical triangle. And the doctrine of ſpherical triangles has the following theorems, which are demonſtrated by the principles of geometry.

345. A right angled ſpherical triangle has its hypothenuſe or longeſt ſide; its perpendicular or ſhorteſt ſide; and its baſe the other ſide.

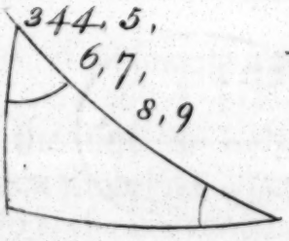
THEOREM.

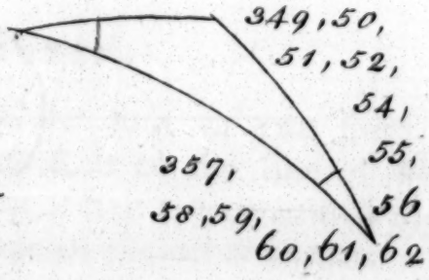
Plane Triangles.

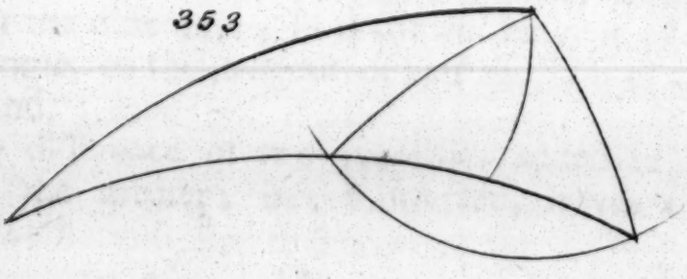




Spherical Triangles.







THEOREM.

346. In a right angled ſpherical triangle; As radius is to the ſine of the angle at the baſe; ſo is the ſine of the hypothenuſe, to the ſine of the perpendicular.

THEOREM.

347. In a right angled ſpherical triangle; As radius is to the coſine of the angle at the baſe; ſo is the tangent of the hypothenuſe, to the tangent of the baſe.

THEOREM.

348. In a right angled ſpherical triangle, rejecting the right angle; either the baſe, or the perpendicular, or coſine of the hypothenuſe, or the coſine of the angle at the baſe, or the coſine of the angle at the perpendicular; either of theſe may be called a middle term, and the two next terms will be neareſt terms; but the other two will be fartheſt terms. And the log. ſine of the middle term augmented by index 10. will make up the log. tangents of the neareſt terms, and the log. coſines of the fartheſt terms.

THEOREM.

349. In all ſpherical triangles, the ſines of the ſides are directly proportional to the ſines of their oppoſite angles.

THEOREM.

350. As the ſine of half the ſum of two ſides of an oblique angled ſpherical triangle, is to the ſine of half the difference; ſo is the co. tangent of half the contained angle, to the tangent of half the difference of the other angles. And,

As the coſine of half the ſum of two ſides, is to the coſine of half the difference; ſo is the co. tangent of half the contained angle, to the tangent of half the ſum of the other angles. And,

Half the difference of two quantities added to half the ſum, gives the greater; but ſubtracted, leaves the leſſer quantity.

THEOREM.

351. As the rectangle of the ſines of any two ſides, is to the ſquare of radius; ſo is the rectangle that is made by the ſines of two arches, one arch being the half ſum of the three ſides, and the other arch being the difference between the third ſide and the half ſum of the three ſides, to a fourth term; which is the ſquare of the coſine of half the angle contained between the two ſides.

352. From this theorem is derived the uſual method of working the azimuth and hour angle, or, which is the ſame thing, having three ſides of the oblique angled ſpherical triangle given, to find one of the angles; thus,

To the complement arithmetical of one of the ſides containing the required angle, add the complement arithmetical of the other of thoſe two ſides; and the ſine of half the ſum of the three ſides, with the ſine of half the ſum of the three ſides leſſened by the ſide oppoſite to the required angle; and half the ſum of theſe four logarithms is the coſine of half the angle required.

THEOREM.

353. As the tangent of half the baſe or longeſt ſide of an oblique angled ſpherical triangle, is to the tangent of half the ſum of the other ſides; ſo is the tangent of half the difference of the other ſides, to the tangent of half the difference of the ſegments of the baſe. The half difference of ſegments added to the half baſe, gives the greater ſegment; but ſubtracted, leaves the leſſer ſegment.

THEOREM.

354. In an oblique angled ſpherical triangle, the ſides may be conſidered as angles, if the angles be conſidered as ſides; and the proportions may be calculated accordingly.

THEOREM.

355. Or, when three ſides of an oblique angled ſpherical triangle are given to find an angle, the ſides including the angle may be called the including ſides, and the other ſide the baſe. And,

As

As the product made of the ſine of the half the ſum of the three ſides, and the ſine of the difference between this half ſum and the baſe :
Is to the ſquare of the radius : :
So is the product made of the ſine of the difference between the two containing ſides ; and the half ſum of the three ſides :
To the ſquare of the tangent of half the angle ſought.

356. OR THUS.

As the product of the ſines of the containing ſides :
Is to the ſquare of the radius : :
So is the product made by the ſine of the difference of the containing ſides, and the ſine of half the ſum of the three ſides :
To the ſquare of the ſine of half the angle ſought.

THEOREM.

357. Or, when the three ſides are given, to find an angle ;
As the product of the ſines of the ſides containing the angle ſought :
Is to the ſquare of the radius : :
So is the difference between the verſed ſine of the ſide oppoſite to the angle ſought ; and the verſed ſine of the arch of difference between the two containing ſides :
To the verſed ſine of the angle ſought.

THEOREM.

358. Or, when three ſides are given, to find an angle ;
Take the difference between the two ſides containing the angle ſought, and ſubtract its verſed ſine from the verſed ſine of the ſide oppoſite to the angle ſought, and divide this remainder by the difference that is made by ſubtracting, the verſed ſine of the ſum of the two ſides including the required angle, from the verſed ſine of the difference of thoſe two ſides ; this laſt remainder is double the verſed ſine of the angle ſought.

THEOREM.

359. When two ſides and the angle between them are given, to find the third ſide ; it will be,

As the cube of the radius,
To the rectangle of the ſines of the containing ſides,
So is the ſquare of the ſine of half the contained angle,
To a fourth term, which is half the difference between the verſed ſine of the third ſide, and the verſed ſine of the difference between the two containing ſides. Therefore, double this half difference, and add it to the verſed ſine of the difference between the two containing ſides, the ſum is the verſed ſine of the ſide ſought.

THEOREM.

360. When two ſides and the contained angle are given, to find the third ſide;

From the verſed ſine of the arch made by the ſum of the two ſides; ſubtract the verſed ſine of the arch made by the difference of thoſe two ſides; take half the remainder, and multiply it into the verſed ſine of the included angle; and add this product to the verſed ſine of the arch of difference between the two ſides; this laſt ſum is the verſed ſine of the third ſide.

THEOREM.

361. When two ſides and the contained angle are given, to find the third ſide;

The product of the ſines of the two containing ſides, being multiplied by the verſed ſine of their included angle; and this product taken from the coſine of the difference of the including ſides; leaves the coſine of the third ſide. But when ſuch ſubtraction cannot be made, taking the coſine from it, leaves the coſine of the third ſide, and the arch is more than 90°.

THEOREM.

362. When two ſides and the contained angle are given, to find the the third ſide;

Take the continued product of the ſines of the including ſides, and the verſed ſine of the included angle; and to this product add the verſed ſine of the arch of difference between the two ſides; the ſum is the verſed ſine of the third ſide.

363. It cannot be expected that the demonſtrations of thoſe theorems ſhould be delived in a treatiſe of practical aſtronomy,

aſtronomy, as they would not only be foreign to the ſubject we are treating of, but ſwell the work beyond the limits to which it is confined. Nevertheleſs, the reader may find them demonſtrated in various trigonometrical writers, ſuch as lord Nepair, Gellibrand, Oughtred, Collins, Caſwell, Norwood, and many others of this nation, as well as by the foreigners, Clavius, Lanſberg, Regiomontanus, Snellius, Pitiſcus, and others of all nations, who have ſpared no pains to diſcover the demonſtrations of thoſe moſt valuable geometrical figures.

364. If H Z O N repreſents the celeſtial meridian of a place, and the arches drawn within it thoſe belonging to the eaſtern hemiſphere, whilſt thoſe belonging to the weſtern hemiſphere are ſuppoſed on the oppoſite ſide; there will be two of thoſe caſes that may happen, one for north latitude, in which the north pole of the equator P is elevated above the horizon, as O P; and the other in which the ſouth pole of the equator is elevated above the horizon, as H S; O repreſenting the north, and H the ſouth point of the horizon itſelf.

365. A variety of ſpherical triangles will be formed in either of theſe four hemiſpheres, namely in the eaſtern hemiſphere for north and ſouth latitude, and in the weſtern hemiſphere for north and ſouth latitude, which triangles are applicable in practical aſtronomy; and in any one of which, any three parts, whether angles or ſides, being given, the other three parts may be found by ſome one or other of the foregoing trigonometrical theorems.

Problem.

366. Given the obliquity of the ecliptic 23° 28′, and the ſun's longitude, Taurus 13° 25′, to find the declination and right aſcenſion?

Calculation.

	°	′		
As ſine — — —	90	0	log.	10 0000000
To ſine longitude from æquator	43	25		9 8371456
So ſine obliquity ecliptic	23	28		9 6001181
To ſine declination — —	15	53		9 4372637

And,

And,

			° ′		
Cosine obliquity ecliptic			23 28	log.	9 9625076
Radius	—	—	90 0		10 0000000
				Sum	19 9625076
Cotangent longitude	—		43 25		10 0240151
Tangent right ascension			40 57½		9 9384925

In the solution of this problem, whilst the sun is in the northern signs between the beginning of Aries, and the beginning of Cancer, the longitude is reckoned in degrees and minutes from Aries, and the right ascension found is reckoned so too. But from the beginning of Cancer to the beginning of Libra, both are reckoned from the beginning of Libra the contrary way. And from the beginning of Libra to the beginning of Capricornus, the reckoning is again successive; afterward it is again reverse.

Problem.

367. Given the angle cCy the latitude 51° 31′ north; cy the sun's north declination 23° 28′. To find Cy the time past 6 before noon, when the sun will be east?

Calculation.

			° ′		
Cotangent angle C		—	51 31	log.	9 9003459
Tangent cy	—	—	23 28		9 6376106
			Sum		19 5379565
			Subtract		10 0000000
	° ′	h ′ ″			
Sine Cy	20 11 or	1 20 44			9 5379565
Add		6 0 0			
Sun east		7 20 44			

And,

			° ′		
As sine angle C	—	—	51 31	log.	9 8935444
To sine cy	—	—	23 28		9 6001181
So sine angle y	—	—	90 0		10 0000000
To sine Cc altitude		—	30 35		9 7065737

In

In the winter months of the year, the ſun will be in the prime vertical, or eaſt and weſt azimuth circle, before ſun riſing, and after ſun ſetting, and therefore cannot be obſerved; otherwiſe this is a moſt uſeful problem in practical aſtronomy, becauſe by correſpondent altitudes taken before and after noon, a meridian line can be drawn, and then, by a clock and the meridian line, it may be known when the ſun is nearly eaſt or weſt, which is the time when he increaſeth faſteſt in altitude, and therefore can be taken moſt advantageouſly for aſcertaining ſolar or equal time.

Problem.

368. Given P O the latitude of a place 51° 31′, and *t* P the complement of the ſun's declination 66° 32′; to find O *t* the complement of the amplitude, or C *t* the amplitude?

Calculation.

	° ′		
Sine of declination —	23 28	log.	9 6001181
Radius — —	90 0		10 0000000
		Sum	19 6001181
Co. ſine latitude —	51 31		9 7939907
Sine amplitude — —	39 47		9 8061274

Which in other words amounts to this proportion.

	° ′		
As coſine latitude —	51 31	log.	9 7939907
To ſine declination —	23 28		9 6001181
So ſine — — —	90 0		10 0000000
To ſine amplitude —	39 47		9 8061274

It appears from the figure, that when the latitude and declination are both of the ſame name, the ſun will riſe and ſet, between the eaſt or weſt, and the point of the horizon under the elevated pole, otherwiſe between the eaſt or weſt, and the point of the horizon over the depreſſed pole.

Had

Had the triangle C x t been uſed, it would have been

			°	′		
As ſine angle C	—		38	29	log.	9 7939907
To ſine t x	—	—	23	28		9 6001181
To ſine angle x	—	—	90	0		10 0000000
To ſine C t	—	—	39	47		9 8061274

PROBLEM.

369. Given the angle t C x the co. latitude 38° 29′ north, and t x the declination 23° 28′ north, to find C x the aſcenſional difference, or time of ſun riſing before 6, and C t the amplitude.

CALCULATION.

			°	′		
Co. tangent angle C	—		38	29	log.	10 0996541
Tangent t x	—	—	23	28		9 6379563
				Sum		19 7376104
				Subtract		10 0000000
Sine C x 33° 8′ or 2^h 12′ 32″						9 7376104
						h ′ ″
Time of ſun riſing before ſix		—				2 12 32
				Add		6 0 0
Time of ſun ſetting	—	—	—			8 12 32
Time of ſun riſing	—	—	—			3 47 28
Length of the day	—	—	—			16 25 4
Length of the night	—	—	—			7 34 56

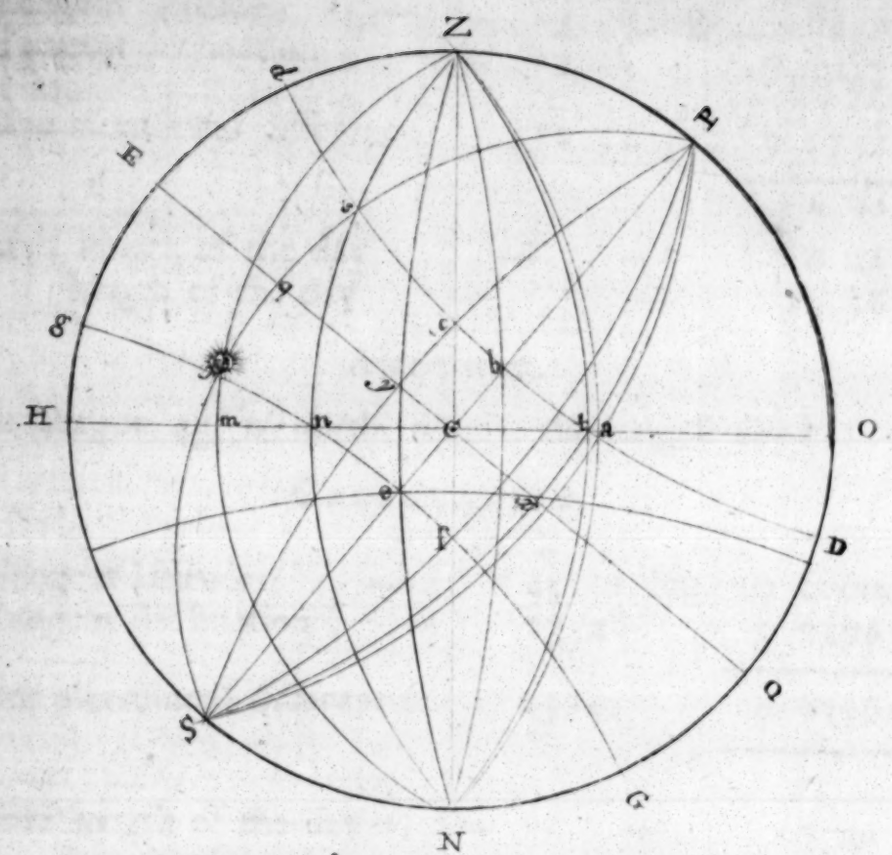
And

			°	′		
As ſine angle C	—		38	29	log.	9 7939907
To ſine t x	—	—	23	28		9 6001181
So ſine angle x	—	—	90	0		10 0000000
To ſine C t	—	—	39	47		9 8061274

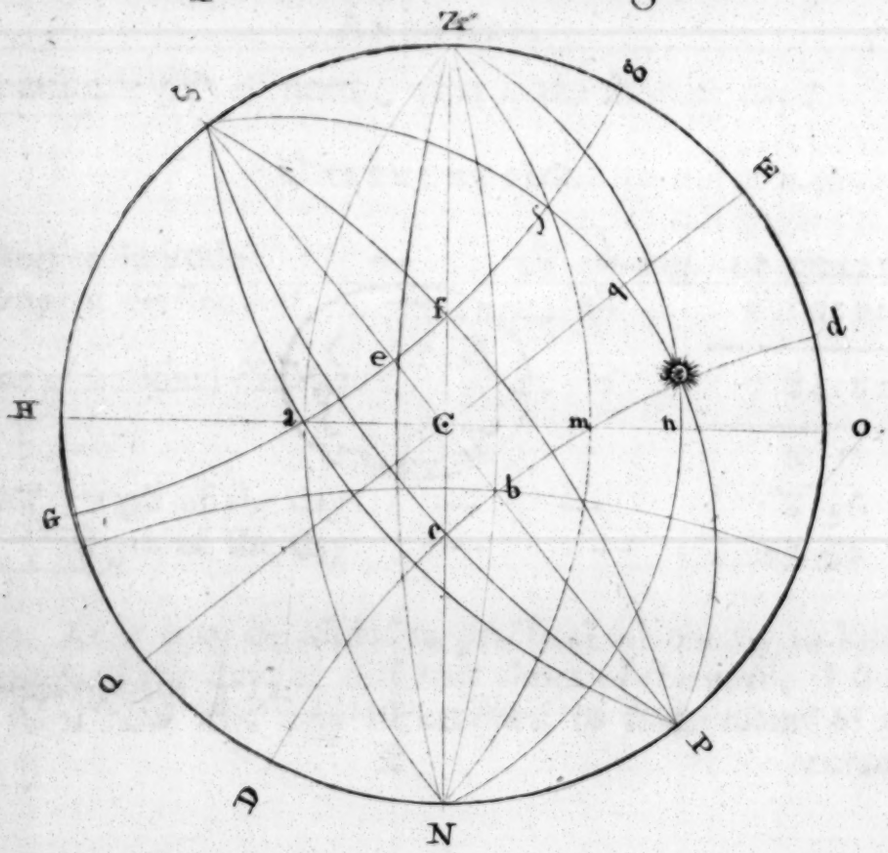
ANOTHER.

In latitude 23° 28′ north, what is the longeſt day, or when the ſun enters Cancer?

CALCU-



Spherical Triangles.



CALCULATION.

		°	′		
Tangent latitude	—	23	28	log.	9 6376106
Tangent declination	—	23	28		9 6376106
Sine aſcenſional difference		10	52		9 2752212
				h	′ ″
Half length of the day	—	—			6 43 28
length of the day	—	—			13 26 56

ANOTHER.

In latitude 45° 0′ north, what is the longeſt day?

CALCULATION.

		°	′		
Tangent latitude	—	45	0	log.	10 0000000
Tangent declination	—	23	28		9 6376106
Sine aſcenſional difference		25	44		9 6376106
				h	′ ″
Half length of the day	—	—			7 42 56
length of the day	—	—			15 25 52

ANOTHER.

In latitude 58° 0′ north, what is the ſhorteſt day?

CALCULATION.

		°	′		
Tangent latitude	—	58	0	log.	10 2042108
Tangent declination	—	23	28		9 6376106
Sine aſcenſional difference		41	0		9 8418214
				h	′
Half length of the day	—	—			8 56
length of the day	—	—			17 52

370. As it may be uſeful in practical aſtronomy to know the length of the day, at different times of the year; I ſhall here ſhew how this may be attained to ſome tolerable accuracy,

curacy, by help of a table for that purpoſe, calculated for the degrees of latitude and the ſun's place in the ecliptic. The uſual tables for this purpoſe are adapted to the ſun's declination, which being much more variable than the apparent motion in the ecliptic, which varies not much unlike the ſucceſſive order of the days of the year, cannot but render this the more acceptable.

371. As the year conſiſts of 365 $\frac{1}{4}$ days, nearly, and the ecliptic has 360 degrees, and every three years following each other, we reckon the year 365 days, and the fourth or Biſſextile year 366 days; it follows that the ſun enters into each ſign of the ecliptic nearly a quarter of a day later every year than the preceding year. And therefore, if the days of the month, when the ſun enters into, and is near the middle of each ſign, be known for a Biſſextile year, the times of entrance into, and being in the middle of the ſigns, for each of the other years, will be nearly known.

372. Thus for the Biſſextile year, the ſun

enters Aries	Mar.	19 $\frac{1}{2}$ nearly	enters Libra	Sep.	22 nearly
is middle	April	3 $\frac{3}{4}$	is middle	Oct.	7 $\frac{1}{4}$
enters Taurus	April	19	enters Scorp.	Oct.	22 $\frac{1}{2}$
is middle	May	4 $\frac{1}{2}$	is middle	Nov.	6 $\frac{1}{2}$
enters Gemini	May	20	enters Sagit.	Nov.	21 $\frac{1}{4}$
is middle	June	5 $\frac{3}{4}$	is middle	Dec.	6
enters Cancer	June	20 $\frac{1}{2}$	enters Capr.	Dec.	20 $\frac{3}{4}$
is middle	July	6 $\frac{1}{4}$	is middle	Jan.	5 $\frac{1}{4}$
enters Leo	July	22	enters Aqu.	Jan.	20
is middle	Aug.	6 $\frac{1}{2}$	is middle	Feb.	3 $\frac{3}{4}$
enters Virgo	Aug.	22 $\frac{3}{4}$	enters Piſces	Feb.	18 $\frac{1}{2}$
is middle	Sep.	6 $\frac{3}{4}$	is middle	Mar.	4 $\frac{1}{2}$

373. From theſe conſiderations and the following table it will be eaſy to know what length the days would be of at different times of the year, and in different latitudes, was there no refraction; or ſomewhat near the truth, though not correctly. Suppoſe, for inſtance, the length of the day in latitude 52° north, for April 25th Biſſextile year, be required. This being 6 days after the entrance into Taurus; look for 6 degrees of Taurus, and latitude 52°; and it gives 7^h 12′ for half the day; ſo the whole day is 14^h 24′.

Had this example been computed, it would have been thus:

Tangent

Tangent latitude —	52°	0′	log.	10 1071902
Tangent declination —	13	32		9 3814655
Sine aſcenſional difference	17	56		9 4886557

	h	′	″
Half length of the day — — —	7	12	8
length of the day — — —	14	24	16

374. When the latitude and ſigns are alike, that is, both north, or both ſouth, the half length of the day is ſhewn; otherwiſe the half length of the night.

When a perſon is on the open ſea, there can be no interruption of the viſible riſing or ſetting of the ſun, by the interpoſition of mountains or hills; but on land, ſuch eminencies may interrupt the appearance of the ſun at riſing or ſetting, when their diſtance is not great, ſome minutes of time. Nevertheleſs, a far diſtant horizon on land, will often be nearly as good as the horizon of the ſea; and therefore may be uſed and applied in this or other caſes accordingly.

375. A TABLE, ſhewing what the half length of the day, and half length of the night, for every degree of latitude from 35° to 58° incluſive, would be, for every degree of the ſun's place in the ecliptic, throughout the year; if the ſurface of the earth and ſea was that of a perfect globe or ſphere, and there was no refraction to anticipate the time of ſun riſing, and retard the time of ſun ſetting: the method of finding the effect of both of theſe, being ſhewn at the end of the table.

Latitude 35°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
o	h	′	h	′	h	′	o
0	6	0	6	33	7	0	30
1		1		34		1	29
2		2		35		1	28
3		3		36		2	27
4		4		37		3	26
5		6		38		4	25
6		7		39		5	24
7		8		40		5	23
8		9		41		6	22
9		10		42		6	21
10		11		43		7	20
11		12		44		9	19
12		13		45		9	18
13		14		46		9	17
14		16		47		9	16
15		17		48		9	15
16		18		49		10	14
17		19		49		10	13
18		20		50		10	12
19		21		51		10	11
20		22		51		10	10
21		23		52		10	9
22		24		53		10	8
23		25		54		11	7
24		26		55		11	6
25		27		56		11	5
26		29		56		11	4
27		30		57		11	3
28		31		58		11	2
29		32		59		11	1
30		33	7	0		11	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 36°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
o	h	′	h	′	h	′	o
0	6	0	6	34	7	2	30
1		1		35		3	29
2		2		36		3	28
3		3		37		4	27
4		4		38		5	26
5		6		39		5	25
6		7		40		6	24
7		8		41		7	23
8		9		42		7	22
9		10		43		8	21
10		11		44		8	20
11		13		45		9	19
12		14		46		9	18
13		15		47		10	17
14		16		48		10	16
15		17		49		11	15
16		18		50		12	14
17		20		51		12	13
18		21		52		12	12
19		22		53		13	11
20		23		54		13	10
21		24		55		13	9
22		25		56		13	8
23		26		56		13	7
24		27		57		13	6
25		28		58		14	5
26		30		59		14	4
27		31	7	0		14	3
28		32		1		14	2
29		33		1		14	1
30		34		2		14	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 37°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
°	h	′	h	′	h	′	°
0	6	0	6	35	7	4	30
1		1		36		5	29
2		3		38		6	28
3		4		39		7	27
4		5		40		8	26
5		6		41		8	25
6		7		42		9	24
7		8		43		9	23
8		10		44		10	22
9		11		45		10	21
10		12		46		11	20
11		13		47		11	19
12		14		48		12	18
13		15		49		12	17
14		17		50		13	16
15		18		51		13	15
16		19		52		13	14
17		21		53		14	13
18		22		54		14	12
19		23		55		14	11
20		24		56		15	10
21		25		57		15	9
22		26		58		15	8
23		27		59		16	7
24		28	7	0		16	6
25		29		1		16	5
26		31		1		16	4
27		32		2		16	3
28		33		3		16	2
29		34		3		17	1
30		35		4		17	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 38°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
°	h	′	h	′	h	′	°
0	6	0	6	37	7	7	30
1		1		38		8	29
2		3		39		8	28
3		4		40		9	27
4		5		41		10	26
5		7		42		10	25
6		8		43		11	24
7		9		44		11	23
8		10		46		12	22
9		11		47		13	21
10		12		48		14	20
11		14		49		14	19
12		15		50		15	18
13		16		51		15	17
14		18		52		16	16
15		19		53		16	15
16		20		54		17	14
17		21		55		17	13
18		22		56		17	12
19		23		57		17	11
20		25		58		18	10
21		26		59		18	9
22		27	7	0		18	8
23		29		1		19	7
24		30		2		19	6
25		31		3		19	5
26		32		3		19	4
27		33		4		19	3
28		34		5		19	2
29		36		6		19	1
30		37		7		19	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 39°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
°	h	′	h	′	h	′	°
0	6	0	6	38	7	9	30
1		1		39		10	29
2		3		40		11	28
3		4		41		12	27
4		5		42		13	26
5		7		44		13	25
6		8		45		14	24
7		9		46		15	23
8		11		47		15	22
9		12		48		16	21
10		13		49		16	20
11		14		51		17	19
12		15		52		17	18
13		16		53		18	17
14		18		54		18	16
15		19		55		19	15
16		20		56		19	14
17		22		57		20	13
18		23		58		20	12
19		24		59		20	11
20		26	7	0		20	10
21		27		1		21	9
22		28		2		21	8
23		30		3		21	7
24		31		4		22	6
25		32		5		22	5
26		33		6		22	4
27		34		7		22	3
28		35		8		22	2
29		37		8		22	1
30		38		9		22	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 40°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
°	h	′	h	′	h	′	°
0	6	0	6	39	7	12	30
1		1		40		13	29
2		3		42		13	28
3		4		43		14	27
4		5		44		15	26
5		7		46		16	25
6		8		47		17	24
7		9		48		18	23
8		11		50		18	22
9		12		51		19	21
10		13		52		19	20
11		15		53		20	19
12		16		54		20	18
13		17		55		21	17
14		19		56		21	16
15		20		57		22	15
16		21		58		22	14
17		23		59		23	13
18		24	7	0		23	12
19		25		1		23	11
20		27		2		24	10
21		28		3		24	9
22		29		4		24	8
23		31		5		25	7
24		32		6		25	6
25		33		7		25	5
26		35		8		25	4
27		36		9		25	3
28		37		10		25	2
29		38		11		26	1
30		39		12		26	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 41°.

°	Aries Libra h	′	Taurus Scorpio h	′	Gemini Sagittar. h	′	°
0	6	0	6	41	7	15	30
1		1		42		16	29
2		3		44		17	28
3		4		45		17	27
4		5		46		18	26
5		7		47		19	25
6		8		48		19	24
7		9		49		20	23
8		11		51		21	22
9		12		52		21	21
10		14		53		22	20
11		15		55		23	19
12		17		56		23	18
13		18		57		24	17
14		20		58		24	16
15		21		59		25	15
16		22	7	0		25	14
17		24		2		26	13
18		25		3		26	12
19		26		4		26	11
20		28		5		27	10
21		29		6		27	9
22		30		7		27	8
23		32		8		28	7
24		33		9		28	6
25		34		10		28	5
26		36		11		29	4
27		37		12		29	3
28		38		13		29	2
29		40		14		30	1
30		41		15		30	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 42°.

°	Aries Libra h	′	Taurus Scorpio h	′	Gemini Sagittar. h	′	°
0	6	0	6	42	7	17	30
1		1		43		18	29
2		3		45		19	28
3		4		46		20	27
4		6		47		21	26
5		8		49		22	25
6		9		50		23	24
7		10		51		24	23
8		12		53		24	22
9		13		54		25	21
10		14		55		26	20
11		16		57		26	19
12		17		58		27	18
13		18		59		27	17
14		20	7	0		28	16
15		21		1		28	15
16		23		2		29	14
17		25		4		29	13
18		26		5		30	12
19		27		6		30	11
20		29		7		30	10
21		30		8		31	9
22		31		9		31	8
23		33		10		31	7
24		34		11		32	6
25		35		13		32	5
26		37		14		32	4
27		38		15		32	3
28		39		16		32	2
29		41		16		32	1
30		42		17		32	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 43°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.	
o	h	′	h	′	h	′	o
0	6	0	6	44	7	20	30
1		1		45		21	29
2		3		47		22	28
3		4		48		23	27
4		6		49		24	26
5		8		51		25	25
6		9		52		26	24
7		10		53		27	23
8		12		55		27	22
9		13		56		28	21
10		15		57		29	20
11		17		59		29	19
12		18	7	0		30	18
13		19		1		31	17
14		21		3		31	16
15		22		4		32	15
16		24		5		32	14
17		26		6		32	13
18		27		7		33	12
19		28		8		33	11
20		30		10		33	10
21		31		11		34	9
22		32		12		34	8
23		34		13		34	7
24		35		14		35	6
25		37		15		35	5
26		39		16		35	4
27		40		17		35	3
28		41		18		35	2
29		43		19		36	1
30		44		20		36	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.	

Latitude 44°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.	
o	h	′	h	′	h	′	o
0	6	0	6	45	7	23	30
1		1		47		24	29
2		4		48		25	28
3		5		50		26	27
4		6		51		27	26
5		8		53		28	25
6		9		54		29	24
7		11		55		30	23
8		13		57		30	22
9		14		58		31	21
10		15		59		32	20
11		17	7	1		32	19
12		18		2		33	18
13		20		3		34	17
14		22		5		34	16
15		23		6		35	15
16		24		7		36	14
17		26		9		36	13
18		27		10		37	12
19		29		11		37	11
20		30		12		37	10
21		32		13		38	9
22		33		14		38	8
23		35		16		38	7
24		36		17		39	6
25		38		18		39	5
26		39		19		39	4
27		41		20		39	3
28		42		21		39	2
29		44		22		39	1
30		45		23		39	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.	

Latitude

Latitude 45°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
°	h	′	h	′	h	′	°
0	6	0	6	47	7	26	30
1		2		48		27	29
2		3		50		28	28
3		5		51		29	27
4		7		53		30	26
5		8		54		31	25
6		10		56		32	24
7		11		57		33	23
8		13		59		34	22
9		14	7	0		35	21
10		16		1		36	20
11		17		3		36	19
12		19		4		37	18
13		21		5		38	17
14		22		7		38	16
15		24		8		39	15
16		26		9		39	14
17		27		11		40	13
18		29		12		40	12
19		30		13		41	11
20		32		15		41	10
21		33		16		41	9
22		35		17		42	8
23		36		19		42	7
24		38		20		42	6
25		40		21		42	5
26		41		22		42	4
27		43		23		43	3
28		44		24		43	2
29		46		25		43	1
30		47		26		43	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 46°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
°	h	′	h	′	h	′	°
0	6	0	6	49	7	30	30
1		2		50		31	29
2		3		52		32	28
3		5		53		33	27
4		7		54		34	26
5		8		56		35	25
6		10		57		36	24
7		12		59		37	23
8		13	7	0		37	22
9		15		2		38	21
10		17		4		39	20
11		18		5		39	19
12		20		7		40	18
13		22		8		41	17
14		23		10		41	16
15		25		11		42	15
16		27		12		43	14
17		28		14		43	13
18		30		15		44	12
19		31		16		44	11
20		33		18		45	10
21		34		19		45	9
22		36		20		45	8
23		37		22		46	7
24		39		23		46	6
25		41		24		46	5
26		42		25		46	4
27		44		26		47	3
28		46		27		47	2
29		47		29		47	1
30		49		30		47	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 47°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
o	h	′	h	′	h	′	o
0	6	0	6	50	7	33	30
1		2		52		34	29
2		3		53		35	28
3		5		55		36	27
4		7		57		37	26
5		8		58		38	25
6		10	7	0		39	24
7		12		2		40	23
8		14		3		41	22
9		15		5		42	21
10		17		6		43	20
11		18		8		43	19
12		20		9		44	18
13		22		10		45	17
14		24		12		45	16
15		26		13		46	15
16		28		15		47	14
17		29		16		47	13
18		31		18		48	12
19		33		19		48	11
20		34		21		48	10
21		36		22		49	9
22		38		23		49	8
23		39		25		49	7
24		41		26		50	6
25		43		27		50	5
26		44		28		50	4
27		46		29		51	3
28		47		30		51	2
29		49		32		51	1
30		50		33		51	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 48°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
o	h	′	h	′	h	′	o
0	6	0	6	52	7	36	30
1		2		54		37	29
2		4		55		39	28
3		6		57		40	27
4		8		59		41	26
5		9	7	0		42	25
6		11		2		43	24
7		13		4		44	23
8		14		5		45	22
9		16		7		46	21
10		18		9		47	20
11		19		10		47	19
12		21		12		48	18
13		23		14		49	17
14		24		15		49	16
15		26		17		50	15
16		28		18		51	14
17		30		20		51	13
18		32		21		52	12
19		34		22		53	11
20		35		24		53	10
21		37		25		54	9
22		39		26		54	8
23		40		28		54	7
24		42		29		55	6
25		44		30		55	5
26		45		32		55	4
27		47		33		56	3
28		49		34		56	2
29		50		35		56	1
30		52		36		56	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 49°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
°	h	′	h	′	h	′	°
0	6	0	6	54	7	40	30
1		2		56		41	29
2		4		57		43	28
3		6		59		44	27
4		8	7	1		45	26
5		9		2		46	25
6		11		4		47	24
7		13		6		48	23
8		14		7		49	22
9		16		9		50	21
10		18		11		51	20
11		20		13		52	19
12		22		15		53	18
13		24		16		54	17
14		25		18		54	16
15		27		19		55	15
16		29		21		56	14
17		31		23		56	13
18		33		24		57	12
19		35		25		57	11
20		36		27		58	10
21		38		28		58	9
22		40		29		58	8
23		42		31		59	7
24		44		32		59	6
25		46		33		59	5
26		47		35	8	0	4
27		49		36		0	3
28		51		37		0	2
29		52		39		0	1
30		54		40		0	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 50°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
°	h	′	h	′	h	′	°
0	6	0	6	56	7	44	30
1		2		58		45	29
2		4		59		47	28
3		6	7	1		48	27
4		8		3		49	26
5		9		5		50	25
6		11		7		51	24
7		13		9		52	23
8		15		10		53	22
9		17		12		54	21
10		19		14		55	20
11		21		15		56	19
12		23		17		57	18
13		25		19		58	17
14		26		20		58	16
15		28		22		59	15
16		30		24	8	0	14
17		32		25		0	13
18		34		27		1	12
19		36		28		2	11
20		38		30		2	10
21		40		31		3	9
22		42		33		3	8
23		43		34		4	7
24		45		36		4	6
25		47		37		4	5
26		49		39		4	4
27		51		40		4	3
28		53		41		4	2
29		54		43		5	1
30		56		44		5	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 51°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
o	h	′	h	′	h	′	o
0	6	0	6	58	7	48	30
1		2		0		49	29
2		4		2		51	28
3		6		4		52	27
4		8		6		53	26
5		10		7		55	25
6		12		9		56	24
7		14		11		57	23
8		16		13		58	22
9		18		15		59	21
10		20		17	8	0	20
11		22		18		1	19
12		24		20		2	18
13		26		22		3	17
14		27		23		3	16
15		29		25		4	15
16		31		27		5	14
17		33		28		5	13
18		35		30		6	12
19		37		32		7	11
20		39		33		7	10
21		41		35		8	9
22		43		36		8	8
23		45		38		8	7
24		47		39		9	6
25		49		41		9	5
26		51		42		9	4
27		53		44		10	3
28		55		45		10	2
29		56		47		10	1
30		58		48		10	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 52°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
o	h	′	h	′	h	′	o
0	6	0	7	0	7	52	30
1		2		2		54	29
2		4		4		55	28
3		6		6		56	27
4		8		8		58	26
5		10		10		59	25
6		12		12	8	0	24
7		14		14		1	23
8		16		16		3	22
9		18		17		4	21
10		20		19		5	20
11		22		21		6	19
12		24		23		7	18
13		25		25		7	17
14		28		27		8	16
15		31		28		9	15
16		33		30		10	14
17		35		32		11	13
18		37		34		11	12
19		39		35		12	11
20		41		37		13	10
21		43		39		13	9
22		45		40		13	8
23		47		42		14	7
24		49		43		14	6
25		51		45		14	5
26		53		47		15	4
27		55		48		15	3
28		57		50		15	2
29		58		51		15	1
30	7	0		52		15	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude

Latitude 53°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
°	h	′	h	′	h	′	°
0	6	0	7	4	7	57	30
1		2		5		58	29
2		4		7	8	0	28
3		6		9		1	27
4		8		11		2	26
5		11		13		4	25
6		13		15		5	24
7		15		17		6	23
8		17		18		8	22
9		19		20		9	21
10		21		22		10	20
11		23		24		11	19
12		25		26		12	18
13		27		28		13	17
14		30		30		14	16
15		32		32		15	15
16		34		34		16	14
17		36		35		17	13
18		38		37		17	12
19		40		39		18	11
20		42		40		19	10
21		43		42		19	9
22		46		44		19	8
23		48		46		20	7
24		50		48		20	6
25		52		49		20	5
26		55		51		21	4
27		57		52		21	3
28		59		54		21	2
29	7	1		55		22	1
30		3		57		22	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 54°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
°	h	′	h	′	h	′	°
0	6	0	7	5	8	2	30
1		2		7		3	29
2		5		9		5	28
3		7		11		6	27
4		9		13		7	26
5		11		15		9	25
6		13		17		10	24
7		15		19		11	23
8		18		22		13	22
9		20		24		14	21
10		22		26		15	20
11		24		28		16	19
12		26		30		17	18
13		28		32		18	17
14		31		33		19	16
15		33		35		20	15
16		35		37		21	14
17		37		39		22	13
18		39		41		23	12
19		41		43		24	11
20		44		45		25	10
21		46		47		25	9
22		48		49		25	8
23		50		50		26	7
24		52		52		26	6
25		54		54		26	5
26		57		55		27	4
27		59		57		27	3
28	7	1		59		27	2
29		3	8	0		27	1
30		5		2		27	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude

Latitude 55°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
o	h	′	h	′	h	′	o
0	6	0	7	8	8	7	30
1		2		10		9	29
2		5		12		10	28
3		7		14		12	27
4		9		16		13	26
5		12		19		15	25
6		14		21		16	24
7		16		23		17	23
8		18		25		19	22
9		20		27		20	21
10		22		29		21	20
11		25		31		22	19
12		27		33		23	18
13		29		35		24	17
14		32		37		25	16
15		34		39		26	15
16		36		41		27	14
17		39		43		28	13
18		41		45		29	12
19		43		47		30	11
20		46		49		30	10
21		48		51		31	9
22		50		53		31	8
23		52		54		32	7
24		54		56		32	6
25		56		58		32	5
26		59	8	0		33	4
27	7	1		2		33	3
28		3		4		33	2
29		6		5		34	1
30		8		7		34	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 56°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
o	h	′	h	′	h	′	o
0	6	0	7	10	8	12	30
1		2		12		14	29
2		5		15		15	28
3		7		17		17	27
4		9		19		19	26
5		12		22		20	25
6		14		24		22	24
7		16		26		23	23
8		19		28		25	22
9		21		30		26	21
10		23		32		27	20
11		26		35		29	19
12		28		37		30	18
13		30		39		31	17
14		33		41		32	16
15		35		43		33	15
16		37		45		34	14
17		40		47		35	13
18		42		49		36	12
19		44		51		37	11
20		47		54		37	10
21		49		56		38	9
22		51		58		38	8
23		54		59		39	7
24		56	8	1		39	6
25		58		3		39	5
26	7	1		5		40	4
27		3		7		40	3
28		5		9		40	2
29		8		10		41	1
30		10		12		41	0
	Virgo Pisces		Leo Aquar.		Cancer Capric.		

Latitude 57°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
°	h	′	h	′	h	′	°
0	6	0	7	13	8	18	30
1		2		15		20	29
2		5		18		21	28
3		7		20		23	27
4		10		22		25	26
5		12		25		26	25
6		15		27		28	24
7		17		29		30	23
8		20		31		31	22
9		22		33		33	21
10		24		36		34	20
11		27		38		36	19
12		29		41		37	18
13		32		43		38	17
14		34		46		39	16
15		37		48		40	15
16		39		50		41	14
17		42		52		42	13
18		44		54		43	12
19		46		56		44	11
20		49		58		44	10
21		51	8	0		45	9
22		54		2		46	8
23		56		5		46	7
24		59		7		47	6
25	7	1		9		47	5
26		4		10		48	4
27		6		12		48	3
28		8		14		48	2
29		11		16		48	1
30		13		18		48	0
	Virgo Piſces		Leo Aquar.		Cancer Capric.		

Latitude 58°.

	Aries Libra		Taurus Scorpio		Gemini Sagittar.		
°	h	′	h	′	h	′	°
0	6	0	7	16	8	24	30
1		3		18		26	29
2		5		21		28	28
3		8		23		30	27
4		10		26		32	26
5		13		28		33	25
6		15		31		35	24
7		18		33		37	23
8		20		36		38	22
9		23		38		40	21
10		26		40		41	20
11		28		43		43	19
12		31		45		44	18
13		33		47		45	17
14		36		50		47	16
15		38		52		48	15
16		41		54		49	14
17		43		57		50	13
18		46		59		51	12
19		48	8	1		52	11
20		51		4		52	10
21		53		6		53	9
22		56		8		54	8
23		58		10		54	7
24	7	1		12		55	6
25		3		14		55	5
26		6		16		56	4
27		8		18		56	3
28		11		20		56	2
29		13		22		56	1
30		16		24		56	0
	Virgo Piſces		Leo Aquar.		Cancer Capric.		

376. Was

376. Was the earth of a globular or truly ſpherical form, and was there no refraction, the time of ſun ſetting, or ſun riſing, would be according to the numbers in this table, ſuppoſing the inſtant of time when the ſun's centre appeared in the horizon to be the time of ſetting or riſing; but as theſe cauſes, and the appearance of the ſun's limb before the centre, may in ſome latitudes lengthen the apparent day 8 or 10 minutes of time, it may be proper to find the quantity of that effect for any of the latitudes; and this may be determined ſeveral ways.

377. If when a clock is brought to keep nearly ſolar time, or equal time, the time of ſun riſing and ſun ſetting be obſerved, and the interval of time compared with the proper numbers of the table, the difference will ſhew the effect of refraction and ſemidiameter, and the half difference of time will ſhew the deception by theſe cauſes. And this method being applied at ſea, will point out the time when the amplitude is to be obſerved.

378. If, for the afternoon, the apparent time of ſun ſetting be when his centre appears in the viſible horizon, his centre having been in the horizon ſooner; whilſt that centre has been paſſing through a perpendicular deſcent of the horizontal refraction, the time of deſcent in the parallel of declination will be enlarged nearly as the coſine of the latitude is to radius, ſo is the horizontal refraction to a fourth term. And it will be farther lengthened nearly as the coſine of the declination is to radius, ſo is the fourth term to the effect on the half day.

Calculation.

Example 1. Suppoſe the effect of refraction only, required for latitude 51° 31′ north, on half the longeſt day?

		°	′		
As coſine —		51	31	log.	9 7939907
To radius — —		90	0		10 0000000
So refraction — —			34		1 5314789
To — —			54,63		1 7374882

And,

And,		° ′		
As cosine	—	23 28	log.	9 9625076
To radius	— —	90 0		10 0000000
So —	54′,63			1 7374882
To —	59′,56 or 4′ of time nearly			1 7749806

CALCULATION.

Example 2. What is the effect in latitude 35° north, when the sun's declination is 20° 0′ north, on half the day?

		° ′		
As cosine	— —	35 0	log	9 9133645
To radius	— —	90 0		10 0000000
So refraction	—	34		1 5314789
To —	—	41′,51		1 6181144
And,		° ′		
As cosine	— —	20 0	log.	9 9729858
To radius	— —	90 0		10 0000000
So —	41′,51			1 6181144
To —	44′,17 or 2′ 57″ of time			1 6451286

CALCULATION.

Example 3. What is the effect in latitude 58° north, on half the longest day?

		° ′		
As cosine	—	58 0	log.	9 7242097
To radius	— —	90 0		10 0000000
So refraction	—	34		1 5314789
To —	—	64′,16		1 8072692
And,		° ′		
As cosine	—	23 28	log.	9 9625076
To radius	— —	90 0		10 0000000
So —	64′,16			1 8072692
To —	69′,94 or 4′ 40″ of time			1 8447616

But in ſuch calculations the horizontal refraction ſhould be nearly known. Theſe proportions may likewiſe be wrought by the parallactic triangle. For if the refraction be found in the lower line C B, and you guide your eye from thence upward, till you come to a radius going to the latitude in the arch; again guiding your eye from that point of interſection by the direction of the concentric circles, to the line C B gives the firſt effect for refraction. And this method is to be obſerved for the coſine of the declination to find the ſecond effect, or for the whole. In theſe three examples it may be ſuppoſed, as though the latitude of the place derived from obſervation is to be applied; but that latitude will be proved farther on, to exceed the true latitude that is to be uſed in regulating altitudes above, or depreſſions below the viſible horizon; and therefore, when that exceſs is known, it muſt be ſubtracted from the obſerved latitude, before this operation is performed, to be as exact as poſſible.

379. As this table ſhews to the neareſt minute of time, excluſive of the effects of refraction and the ſpheroidal figure of the earth, it may be ſuppoſed that, by the foregoing method of making the experiment, the time-keeper may be ſet by this method ſo as to determine the meridian to half a minute of time by inſpection only, where the obſerver has an open and well-extended horizon for the purpoſe. And therefore this method may be uſeful at ſea, for taking the variation of the needle, at or very near noon, in high latitudes; and in taking the latitude by the moon, when her declination is known, and allowance is made for her parallax in altitude.

380. On land this table may be uſeful to young practical aſtronomers throughout England, Scotland, and Ireland; almoſt all parts of Europe between the Baltic Sea and the Mediterranean; to all parts of North America, from Carolina to New England and Hudſon's Bay; and to other places North and South of the line, within the limits of the table; which parts of the earth and ſeas may be ſeen on the map or chart.

381. The uſe and application of nice aſtronomical inſtruments has been hitherto omitted to be treated of in this work, with a deſign to ſhew firſt what can be done by the leaſt

leaſt complicated ones, and ſometimes by none, but always with inſtruments or helps that are not difficult to be had. Now a ſtep farther is to be taken, to ſhew what errors ariſe in ſome practices at ſea with the beſt angular inſtruments, in not allowing for the ſpheroidal figure of the waters of the ocean, and the change of the curvature near the horizon that is produced by it.

382. It is twenty-three years ſince I threw out an hint concerning this in the Gentleman's Magazine, 1751, page 360, dated June 21, in theſe words: "*Mr. Urban*, It is "an opinion generally received amongſt the beſt mariners, "that by Mr. Hadley's octant and other inſtruments mi- "nutely graduated, and which uſe the viſible horizon, "the ſun's meridian altitude, and thereby the latitude, may "be taken to a minute at ſea. But this cannot be; for, "by Prop. 20. Book iii. of Sir Iſaac Newton's Principia, "the equatorial diameter is thirty-four miles longer than the "polar axis." Then follows the objection, and a method of correcting the error in latitude, which at that time I made $8\frac{1}{2}$ miles; and noting at the concluſion, that this error of latitude ariſing from the ſpheroidal figure of the earth might be corrected, if, by any artifice, the north and ſouth points of the horizon could be avoided, and obſervations could be made under an eaſt or weſt direction, or both, for the latitude; of which there are ſeveral methods, one by help of correſpondent altitudes and the true elapſed time. Signed "S. Dunn."

383. It is ten years ſince I extended this plan, and, by a pamphlet printed in April, 1765, and inſcribed to the Honourable Commiſſioners of Longitude, ſhewed that the fallacy in aſcertaining time by the uſual method of obſerva- tion at ſea, could not amount to leſs than half a degree of longitude without an introduction of this correction in uſual caſes. The concluſion of that pamphlet, page 36, is thus: "It hath been hitherto taken for granted, that if a clock, "or any machine, could be made to keep equal time at ſea, "or if the immerſions and emerſions of Jupiter's ſatellites "could be exactly computed and obſerved at ſea, and that "the hour, minute, and ſecond of the day or night could be "exactly taken at ſea, that then the longitude of a ſhip at "ſea might be exactly determined by either of thoſe me-

" thods. But I have demonſtratively ſhewn in this tract, " that if either of thoſe methods do err but one mile when " the corrections here propoſed are applied, that ſame me- " thod will err in many caſes without thoſe corrections " thirty-one miles. And that if either of thoſe methods do " err but ten miles when the corrections here propoſed are " applied, that ſame method will err in many caſes without " thoſe corrections forty miles. And that if either of thoſe " methods do err but thirty miles when the corrections " here propoſed are applied, that ſame method will err " in many caſes without thoſe corrections ſixty miles. And " that if either of thoſe methods do err but ſixty miles " when thoſe corrections here propoſed are applied, that " ſame method will err in many caſes without thoſe correc- " tions ninety miles." And ſoon after I made a ſett of tables, ſhewing, for three different ratios of the equatorial diameter to the polar axis, the deviation of a perpendicular to the horizon, from a direction towards the earth's centre for all latitudes from the equinoctial to the poles. This I offered immediately to publiſh, but was given to underſtand, no advantages were to be expected from it; and therefore it was ſuppreſt.

384. Long before thoſe times, and during the interval, aſtronomers have been ſilent concerning this ſubject. Some of the hair ſplitting profeſſion have deigned to call it the effect of imagination, although a matter of ſcience, ſupported by the cleareſt evidence, and by undeniable facts and experiments. So the immortal Harvey, firſt diſcoverer of the circulation of the blood, was ludicrouſly named Circulator. The time of the viſible riſing and ſetting of the ſun; the length of the day; the amplitude, and conſequently the variation of the magnetic needle at ſea, as determined by the amplitude; the azimuth, and the variation of the needle, as determined by the azimuth: theſe are in ſome meaſure affected by the horizon's being formed by tangents to a ſpheroidal, and not a ſpherical, earth. And a correction of a degree, or half a degree in uſual caſes, in aſcertaining the variation of the needle, is no ſmall correction; nor the correction of the hour angle from 1, to 2, 3, or 4 minutes of time, a ſmall correction as

ſome

ſome practical aſtronomers have been pleaſed wantonly to repreſent it.

385. Were there no other proofs of the truth and certainty of theſe things than what may be inferred from obſervations and experiments judiciouſly made, the effect of this deviation cannot but appear manifeſt to the ſenſes of an aſtronomical enquirer. It is fourteen years ſince I diſcovered its effects myſelf, and, in the interval ſince that time, have left many original obſervations in the hands of other perſons, who might have readily drawn the concluſions, had they been diſpoſſeſſed of prejudice. An inſtance or two may be given.

In the year 1763, having drawn a meridian line by altitudes taken near the prime vertical; February 29, 1764, the ſun was obſerved to tranſit that meridian at $11^h\ 55'\ 10''$, the clock being nearly adjuſted to keep time with the ſun. That morning, at $8^h\ 33'$ per clock, the ſun's altitude was taken $16°\ 48'\ 30$, which gave the hour angle from noon $3^h\ 21'\ 36''$, but it was ſhort of the meridian tranſit $3^h\ 22'\ 6''$; ſo the hour angle by the obſerved altitude was $30''$ too little.

The ſame morning, at $8^h\ 38'$ per clock, the altitude was $17°\ 26'$, from which the hour angle per the altitude, was $3^h\ 16'\ 26''$, but per clock $3^h\ 17'\ 6''$; ſo the hour angle by this obſerved altitude was $40''$ too little.

Had I allowed $10'$ of a degree for the deviation, the former of theſe hour angles would have come out $3^h\ 22'\ 8''$, and thereby the time of the meridional tranſit $11^h\ 55'\ 8''$. And allowing the ſame deviation at the latter altitude, the hour angle would have come out $3^h\ 7'\ 18''$, and thereby the meridional tranſit $11^h\ 55'\ 18''$.

In many other obſervations of this kind, I have found the meridian tranſit anticipated from 38 to 40 ſeconds of time; in others retarded; accordingly as the affected part of the horizon has been depended on: and it is many years ſince I was clearly of opinion, that this would be one of the beſt and moſt effectual methods of aſſigning the ratio of the equatorial diameter to the polar axis, if the experiments and obſervations were made with propriety.

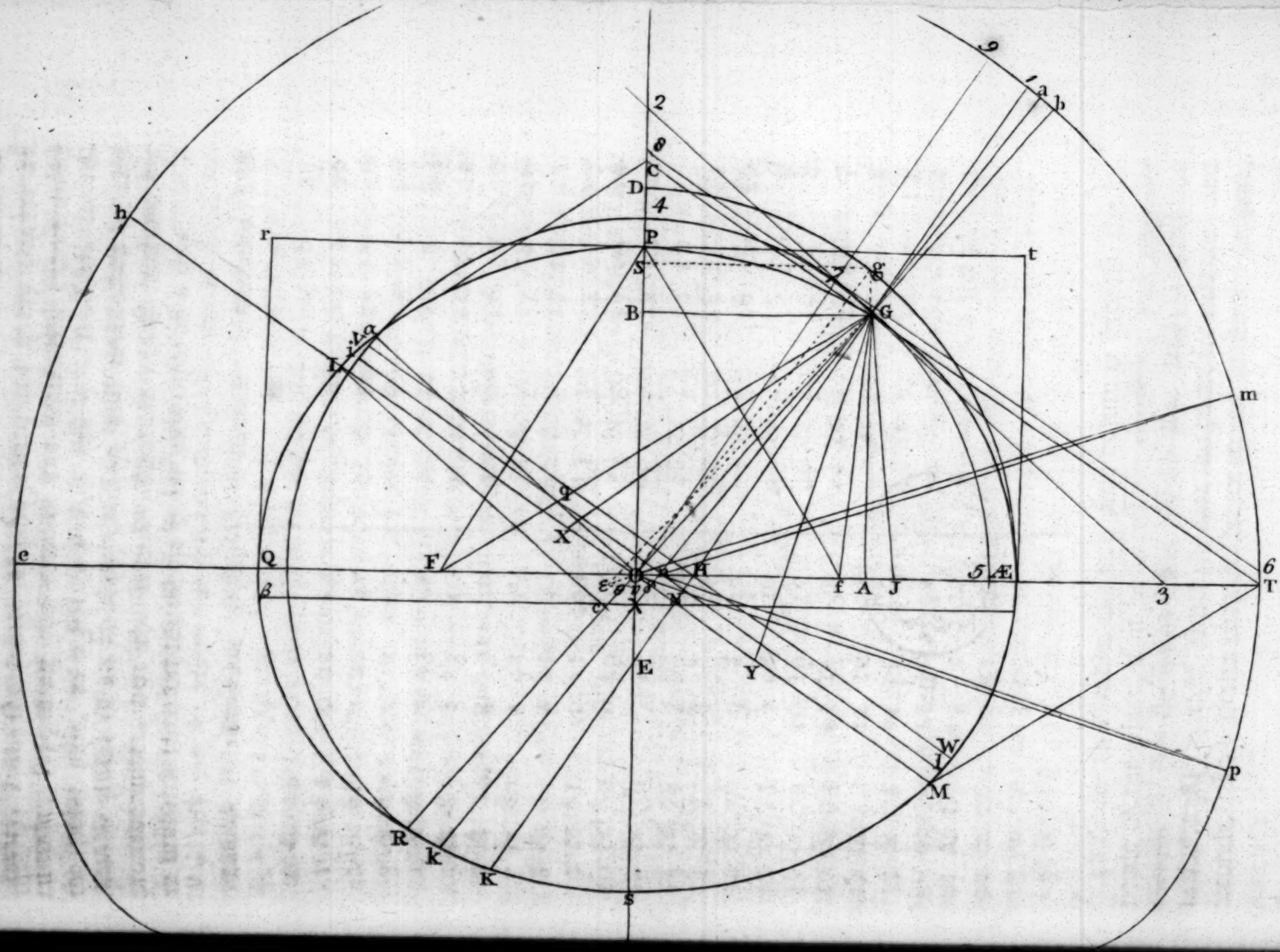
But there is another cauſe that may partly deſtroy that regularity which the deviation would have, the uneven curvature of the lands. This the reader may ſee more of, in my

New

New Atlas of the Mundane Syſtem, or of Geography and Coſmography; lately publiſhed by Mr. Sayer, in Fleet-ſtreet, London; the maps of which, being moſt elegantly executed, may not be improper to be conſulted on ſome occaſions, by perſons who are deſirous of a more particular deſcription of places than is exhibited in this work. I cannot anſwer why aſtronomers have neglected to introduce this deflection of gravity in the computation of their obſervations, ſeeing in many caſes its effects introduce an error of as much as would be introduced in the longitude problem, by an error of one or two minutes of a degree in the moon's place in the heavens; and it makes no difference, as to utility, whether the moon's place be corrected a minute of a degree, or any other correction be introduced adequate thereto. The introduction of corrections for other niceties cannot correct this capital error; and if aſtronomers will ſtill perſiſt in neglecting this correction, it may be ſaid to them, —" hæc oportuit facere, & illa non omittere." Matt. xxiii. 23.

After conſidering what the ratio of the equatorial diameter to the polar axis might be, and deducing a theorem, I made a table, which was nearly as follows:

386. The diagram belonging to the ſubject is P Æ S Q, repreſenting the ſpheroidal figure of the earth flattened at the poles P and S, from which, by conſequence, a perpendicular to any place on the horizon, either on land or at ſea, will not be drawn to O the earth's centre, but to ſome other point as B, where it will cut the equatorial diameter Æ Q, and thereby make an angle with the right line drawn to the centre O, as the angle B G O; this I call the deviation of the direction of gravity from the centre of the earth O.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387. A TABLE, shewing the deviation of the direction of gravity, from the centre of the earth, for every degree of latitude; supposing the ratio of the equatorial diameter to the polar axis, as 230 to $228\frac{7}{10}$ nearly And the terraqueous globe of an uniform density, and unaffected by any protuberances, but the regular figure it would acquire by an uniform rotation round its polar axis once in a day.

Latitude	Deviation	Diff.	Latitude	Deviation	Diff.
° & °	′ ″	″	° & °	′ ″	″
0 & 90	0 0		24 & 66	14 26	27
1 & 89	0 41	41	25 & 65	14 52	26
2 & 88	1 22	41	26 & 64	15 18	26
3 & 87	2 2	40	27 & 63	15 43	25
4 & 86	2 43	41	28 & 62	16 7	24
5 & 85	3 23	40	29 & 61	16 29	22
6 & 84	4 3	40	30 & 60	16 49	20
7 & 83	4 42	39	31 & 59	17 8	19
8 & 82	5 22	40	32 & 58	17 26	18
9 & 81	6 1	39	33 & 57	17 43	17
10 & 80	6 39	38	34 & 56	17 59	16
11 & 79	7 17	38	35 & 55	18 14	15
12 & 78	7 55	38	36 & 54	18 27	13
13 & 77	8 32	37	37 & 53	18 38	11
14 & 76	9 8	36	38 & 52	18 48	10
15 & 75	9 44	36	39 & 51	18 57	9
16 & 74	10 19	35	40 & 50	19 5	8
17 & 73	10 53	34	41 & 49	19 12	7
18 & 72	11 26	33	42 & 48	19 17	5
19 & 71	11 58	32	43 & 47	19 20	3
20 & 70	12 30	32	44 & 46	19 22	2
21 & 69	13 1	31	45	19 23	1
22 & 68	13 31	30			
23 & 67	13 59	28			
$23\frac{1}{2}$ & $66\frac{1}{2}$	14 12				

388. In this table, the two first columns express the latitude of the place, if it be in whole degrees; and opposite thereto is the deviation of a plumb line from a direction towards the earth's centre in that latitude, in minutes and seconds. But if the latitude is not given in whole degrees, the

the column of differences ſhews how much the difference is in a degree of latitude, and conſequently any proportional part, for a part of a degree of latitude may be taken out almoſt by inſpection; and this being either added to, or ſubtracted from, the deviation for the next greater or leſs degree of latitude, as the caſe requires, gives the deviation for the place required. And thus by the table,

		°	′		′	″
The deviation for latitude	—	5	0	is	3	23
The deviation for latitude	—	15	0	is	9	44
The deviation for latitude	—	25	0	is	14	52
The deviation for latitude	—	31	0	is	17	8
The deviation for latitude	—	25	30	is	15	5
The deviation for latitude	—	51	30	is	18	52
The deviation for latitude	—	34	25	is	18	5
The deviation for latitude	—	23	28	is	18	6
The deviation for latitude	—	66	32	is	14	13
The deviation for latitude	—	38	29	is	18	52
The deviation for latitude	—	57	15	is	17	54
The deviation for latitude	—	0 or 90		is	nothing	

389. The theorem by which theſe numbers were calculated was deduced by me from the properties of the conic ſections: as it would be deviating from the practice of aſtronomy, to introduce its demonſtration here, it is therefore omitted; but in words it is thus:

Theorem.

Add together the logarithm of the ſum of the equatorial diameter and polar axis, the logarithm of the difference between the equatorial diameter and polar axis; the logarithm of the ſine of the latitude, and the logarithm of the coſine of the latitude; and from the ſum of theſe four logarithms, ſubtract the double logarithm of the polar axis: the remainder is the logarithm of the ſine of the angle of deviation from the centre of the earth.

Example.

What is the deviation for the latitude of 20°? Suppoſing the ratio of the equatorial diameter to the polar axis as 656248 to 652560, or as 6562 to 6525 nearly?

OPERATION.

Sum — —		13088	log.	4	1168401
Difference —	° ′	37		1	5667909
Sine —	20 0			9	5340517
Coſine —	20 0			9	9729858
		Sum		25	1906685
	6525 its double		log.	7	6292408
Deviation —	12′ 30″		log.	7	5614277

EXAMPLE.

What is the deviation for latitude 51°?

OPERATION.

Two logarithms as before	° ′			5	6836310
Sine — —	51 0		log.	9	8905026
Coſine — —	51 0			9	7988718
		Sum		25	3730054
	6525 its double		log.	7	6292408
Deviation ——	19′ 1″		log.	7	7437646

EXAMPLE.

What is the deviation for latitude 70° 0′?

OPERATION.

Two logarithms as before	° ′			5	6836310
Sine — —	70 0		log.	9	9729858
Coſine — —	70 0			9	5340517
		Sum		25	1906685
	6525 its double		log.	7	6292408
Deviation ——	12′ 30″		log.	7	5614277

Theſe numbers being a little different from thoſe uſed for the table, may make a ſmall diſagreement. But it may be obſerved, that ſeeing the ſine and coſine are always added by the theorem, the deviation muſt be the ſame, for the complements of the latitudes, as for the latitudes themſelves.

390. After I had inveſtigated the above theorem and formed the table, I found that another theorem for this purpoſe might be inveſtigated from other principles; and it readily came out thus.

THEOREM.

As the ſquare of the polar axis :
To the ſquare of the equatorial diameter : :
So is the ſine of the latitude:
To a fourth number.
And,
As the coſine of the latitude :
Is to this fourth number : :
So is radius:
To the tangent of an angle. Which angle exceeds the latitude, by the deviation of the direction of gravity from the earth's centre.

391. In this theorem, aſſuming the two diameters of the earth nearly as before, the ſquare of the leſs to the ſquare of the greater comes out nearly as 1 to 1,007 whoſe logarithm 0,0048952 with the addition of 10 to the index, is a conſtant logarithm. Hence the theorem becomes thus,

THEOREM.

To the log. ſine of the latitude, add the conſtant logarithm 10,0048952, and from the ſum, ſubtract the log. coſine of the latitude, the remainder is the tangent of an arch, which exceeds the latitude by the deviation required.

392. This theorem is likewiſe derived from the properties of the ellipſis, one of the conic ſections; and, as the demonſtration is ſomewhat ſhorter than the former, it may be delivered here.

If T G C be a tangent line drawn north and ſouth, and touching the earth's ſurface at G, and G H E K be a perpendicular thereto cutting the tranſverſe diameter of the ellipſis in H, and the conjugate thereof in E, O being the centre; and if G B be perpendicular to P S; then, as the ſquare of the conjugate diameter, to the ſquare of the tranſverſe diameter, ſo is B O to B E. And, as G B to B E, ſo is radius to tangent of the angle B G E. But B O and B G are nearly the ſine and coſine of the latitude, and B O nearly the tangent of the latitude to the radius B G; therefore

fore B E is the tangent of an angle to the radius B G, which angle exceeds the latitude, by the angle O G E, the angle of deviation required.

Hence the theorem is manifeſt. For the ratio of the two axes of the earth, form the conſtant logarithm, and the reſt depends on the ſines, coſines, and tangents.

This may be illuſtrated by other examples.

Example.

393. What is the deviation for latitude 20° 0′?

Operation.

Sine — —	20° 0′	log.	9 5340517
Conſtant logarithm			10 0048952
		Sum	19 5389469
Coſine — —	20 0	log.	9 9729858
Tangent —	20 12		9 5559611

Hence 12′ 0″ is the deviation.

Example.

What is the deviation for latitude 51° 0′?

Operation.

Sine — —	51° 0′ 0″	log.	9 8905026
Conſtant logarithm			10 0048952
		Sum	19 8953978
Coſine — —	51 0	log.	9 7988718
Tangent —	51 19 0		10 0965260

Hence 19′ 0″ is the deviation.

Example.

What is the deviation for latitude 70° 0′?

OPERATION.

Sine — —	70° 0′ 0″	log.	9	9729858
Conſtant logarithm			10	0048952
		Sum	19	9778810
Coſine — —	70 0	log.	9	5340517
Tangent —	70 12 30		10	4438293

Hence 12′ 30″ is the deviation required.

EXAMPLE.

What is the deviation for latitude 30° 0′ ?

OPERATION.

Sine — —	30° 0′ 0″	log.	9	6989700
Conſtant logarithm	—	—	10	0048952
		Sum	19	7038652
Coſine — —	30 0	log.	9	9375306
Tangent —	30 17 0		9	7663346

Hence 17′ 0″ is the deviation.

EXAMPLE.

What is the deviation for latitude 60° 0′ ?

OPERATION.

Sine — —	60° 0′	log.	9	9375306
Conſtant logarithm			10	0048952
		Sum	19	9424258
Coſine —	60 0	log.	9	6989700
Tangent —	60 17 nearly		10	2434558

Hence 17′ nearly is the deviation.

Hence the deviation comes out the ſame for the latitude, as for the complement of the latitude reckoned from the equinoctial

equinoctial line, by both of theſe theorems, excepting a little difference reſulting from the nature of the logarithmic numbers.

394. In my forementioned calculations on this ſubject, the proceſs in general was made by the natural numbers, ſines, and tangents, &c. to take off any errors that might be apt to be introduced by the few places to which the logarithmic numbers do extend. The lengths of the ſeveral lines more immediately relative to this ſubject, ſuppoſing a perpendicular to be drawn downward from the latitude of London, were as follows:

Q Æ the equatorial diameter — —	6562480
P S the polar axis — — —	6525600
O G the central diſtance — —	3269958½
A G the ſine of the latitude —	2553494
A O the coſine of the latitude —	2042620
O H the perp. diſtance in Q Æ —	22894
O E the perp. diſtance in P S —	28944½

	°	′	″
Whence the angle of deviation O G H	0	18	53
So the apparent latitude was —	51	30	0
And the true latitude was but ——	51	21	7

For was the earth, inſtead of the figure of an oblate ſpheroid flat toward the poles, in the form of a perfect ſphere, the ſame horizon would preſent itſelf for the latitude of 51° 21′ 7″ under that ſpherical figure of the earth, which now preſents itſelf for the latitude of 51° 30′ 0″ under the oblate ſpheroïdal form of the earth.

395. In theſe examples, the ratio of the equatorial diameter to the polar axis has been ſuppoſed nearly as 230 to $228\frac{7}{10}$. And this is but a little different from the proportion of thoſe two diameters to each other, as verified by experiments. Sir Iſaac Newton gives the proportion as 230 to 229. But, by taking the medium of ſeveral others, the proportion of the equatorial diameter to the polar axis is as 230 to $228\frac{2}{3}$. Wherefore if we take Sir Iſaac's numbers, the conſtant logarithm beforementioned will be 10,0057846, And if we take the latter proportion derived from a medium, the conſtant logarithm will be nearly what has been here applied; either of which conſtant logarithms being uſed as the logarithm 10,0048952 was uſed, will give the deviation

tion according to those respective proportions of the equatorial diameter to the polar axis. The effect of this deviation from the centre of the earth, in respect to the latitudes of places and other astronomical observations, but chiefly as it introduces an error in computing the time, will be shewn farther on.

396. The astronomers, who have measured a degree of the meridian in different latitudes, have performed this problem by taking the meridional zenith distances of certain fixed stars, such as have been out of the danger of refraction, and measuring a line as long and as near the meridian as possible; one of the places of observation being in a greater latitude than the other. In this case, from the principles here laid down, the plumb lines, by which their instruments were regulated, could not at both of the places of observation tend towards the same centre.

397. The celebrated measure of M. Picard, from Paris to Amiens, which was performed near a century ago, was repeated, in 1739, by another set of academicians, and the latitudes of the two places were nearly 48° 50′ and 49° 54′, the degree of latitude being by the former measurement 57061 toises, by the latter 57183. The difference of deviation at these two places, as being near the middle of the quadrant, amounts to but a few seconds. Thus,

Sine — —	48° 50′	log.	9 8766-85
Constant logarithm			10 0037846
		Sum	19 8804631
Cosine — —	48 50	log.	9 8183919
Tangent — —	49 4 50		10 0620712

And,	° ′ ″		
Sine — —	49 54	log.	9 8836168
Constant logarithm			10 0037846
		Sum	19 8874014
Cosine —	49 54	log.	9 8089692
Tangent —	50 8 45		10 0784322

Deviation

		°	′	′	″
Deviation in latitude	48	50	is	14	50
Deviation in latitude	49	50	is	14	45
Difference					5

398. The mathematicians, who at the ſame time went to Peru, are ſaid to have meaſured the three firſt degrees of the ſouthern hemiſphere.

			°	′	″		
Sine — —			3	0		log.	8 7188002
Conſtant logarithm							10 0037846
						Sum	18 7225848
Coſine —			3	0		log.	9 9994044
Tangent —			3	1	34		8 7231804
						′	″
Deviation at the equinoctial						0	0
at ſouth bounds			°		′	1	34
on			3		0	1	34
on			1		0		31

OTHERWISE.

			°	′	″		
Sine — —			1	30		log.	8 4179190
Conſtant logarithm							10 0037846
						Sum	18 4217036
Coſine — —			1	30		log.	9 9998512
Tangent — —			1	30	46		8 4218524

						°	′ ″
Deviation on						1	30 is 46
on						1	0 is 31

Or taking the other conſtant logarithm, the computation will be thus:

			°	′	″		
Sine — —			1	30		log.	8 4179190
Conſtant logarithm							10 0048952
						Sum	18 4228142
Coſine — —			1	30		log.	9 9998512
Tangent —			1	31	1		8 4229630

Deviation

Deviation on	—				° 1	′ 30	″ is 61
on	—				1	0	is 41

Or thus,

Sine — —	° 3	′ 0	″	log.	8	7188002
Conſtant logarithm					10	0048952
				Sum	18	7236954
Coſine — —	3	0		log.	9	9994044
Tangent —	3	2	2		8	7242910

Deviation at the equinoctial		′ 0	″ 0
in latitude	3°	2	2
Difference		2	2
on	1°	is	41

399. The latitude of Tornea, near the Gulf of Bothnia, where the mathematicians meaſured in 1736, is 65° 51′ nearly, and northward one degree towards Kittis, where they made another obſervation, the plumb line may be ſuppoſed to have deviated from a direction towards the earth's centre leſs than it deviated at Tornea 22″; which may be found thus:

Sine — —	° 65	′ 51	″	log.	9	9602222
Conſtant logarithm					10	0037846
				Sum	19	9640068
Coſine — —	65	51		log.	9	6118580
Tangent — —	66	2	9		10	3521488
Sine — —	° 66	′ 51	″	log.	9	9635417
Conſtant logarithm					10	0037846
				Sum	19	9635417
Coſine — —	66	51		log.	9	5945469
Tangent — —	67	1	47		10	3727794

Deviation

		′	″
Deviation at Tornea		11	9
near Kittis		10	47
Difference			22

Or taking the other conſtant logarithm; thus,

Sine — —	65° 51′ ″	log.	9	9602222
Conſtant logarithm			10	0048952
		Sum	19	9651174
Coſine — —	65 51	log.	9	6118580
Tangent — —	66 5 24		10	3532594
Sine — —	66° 51′ ″	log.	9	9635417
Conſtant logarithm			10	0048952
		Sum	19	9684369
Coſine — —	66 51	log.	9	5945469
Tangent — —	67 4 57		10	3738900

	′	″
Deviation at Tornea	14	24
near Kittis	13	57
Difference		27

400. In the meaſurement of a degree of latitude by theſe and other perſons, we find no allowance for theſe deviations; therefore whatever ſuch deviations have amounted to, they have been included in the meaſurements, and no accurate length of a degree of latitude has been yet determined; or, what amounts to the ſame thing, an arch of a meridian included between two right lines drawn from two points, a degree diſtant from each other in a meridian of the heavens, to the centre of the earth.

401. In order to know what effect this deviation has on obſervations made by help of the horizon of the ſea, it will be proper to conſider how the apparent or viſible horizon of the ſea is formed, from an indefinite number of tangent lines drawn to a point on the ſurface of the ocean; ſup-

poſing that ocean perfectly ſmooth, and being a part of the ſurface of the oblate ſpheroïd, like the figure of the earth.

402. In the figure, let O Æ be the ſemi-equatorial diameter, O P the ſemi-polar axis, P Æ an elliptical quadrant of a meridian, G a place in that meridian; and having drawn 4 G 5 a quadrant of a circle with the radius O G, let 2 G 3 be a tangent to that circular quadrant, and C G T a tangent to the elliptical one. And let O G *b* be directed towards the zenith from the circular quadrant, but N G 1 be directed towards the zenith from the elliptical quadrant.

403. Then is T 1 the zenith diſtance of the equator or latitude of the place as derived from obſervation, and G Æ, or, which is the ſame thing, the angle G H Æ equal thereto. And drawing a line as 6, 7, 8, parallel to C G T, ſo that it may touch the circular quadrant 4, G, 5, in the point 7, and from thence drawing a line to the centre as 7, O; then 5, 7, or Æ, 7, or, which is the ſame thing, the angle 7, O, Æ, will be the latitude of the place in the ſuppoſed circular meridian, having the ſame horizon 6, 7, 8, as the place G in the elliptical meridian has, namely, C G T; becauſe C G T and 6, 7, 8 are parallel to each other, and here is no parallax to alter the caſe. Conſequently a ſpectator at G has the ſame apparent horizon, being on an elliptical meridian, as another has at 7 on a ſpherical meridian.

404. Furthermore, becauſe the apparent riſing, elevation, depreſſion, and ſetting of the celeſtial bodies, is produced by the rotation of the earth round its axis once in a day, and by an uniform motion from weſt towards the eaſt, whilſt the circles of the heavens at equal diſtances from the centre of the apparent diurnal motion, indefinitely great, do remain fixed and immoveable; and ſeeing that all ſpherical triangles have their angular points in the great circles which are formed in the concave expanſe of the apparently ſpherical heavens, of which the horizons of places on the ſurface of the terreſtrial ball, are circular baſes, on which the altitudes of the ſun and ſtars ſtand and wholly depend: it therefore follows, that neither the hour angle, nor the azimuth angle of a ſpherical triangle, formed in the concave hemiſphere of the heavens, can poſſibly ſtand on any other baſe in the heavens; but, the one on the equator, and the other on the horizon of a place as ſuppoſedly formed by the circumference of a great circle of the heavens, whoſe plane lyeth at contact on the

ſurface

ſurface of a ſuppoſedly ſpherical earth; or, what amounts to the ſame thing, that paſſeth through the earth's centre. For when the latitude of a place is determined by aſtronomical obſervations, it is the zenith diſtance from the equator, or elevation of the pole above the viſible horizon, and that horizon is ſuppoſedly a circular plane, lying at contact on the ſurface of a ſphere or globe; and the circumference of this horizon is the baſe on which all the triangles meeting at the zenith ſtand. In this conſtruction of the triangles, the whole earth is conſidered as no other than a point, and the place of obſervation, the centre of that circular plane or of that horizon on which the triangles ſtand, and from which they are formed; was that centre removed the horizon would likewiſe be removed, and another horizon would be formed, and other triangles would ſtand on this new formed horizon, whilſt they meet at the ſame zenith. But, what is ſtrictly a property of a ſpherical earth, which is indeterminately ſmall compared with the expanſe of the heavens, under whatever direction a perſon walks on the land, or ſails on the ocean, the degrees in the heavens are elevated in the direction he goes, and depreſſed in the contrary direction, as the degrees of diſtance on the ſurface of the ſpherical earth are paſſed over; and this produceth the ſame phænomena as would be produced, was a ſpectator ſituated on an horizon that paſſed through the centre of the ſpherical earth, and that horizon revolved round that centre. Hence ſpherical triangles are formed above and below the horizons of places, and the one are complements to the other. This may be illuſtrated as follows: let a circular plane of any determinate extent, be ſuppoſed to lay at contact on the circular meridian 4 G, 5, at the point G, then will this plane when produced to the heavens, point out the circumference of the horizon of that place; and becauſe of the ſmallneſs of the earth's radius being compared with the extent of the heavens, it will be alike whether that horizon cuts the earth's centre, or lays at contact on its ſurface. The like will follow for horizons formed at other places, and when they are conſidered as formed on the ſurface of a ſpherical earth, vertical triangles may be formed on ſuch horizons, after the ſame manner as they would be formed on the circumferences of other circles equal in diameter, whoſe com-

mon centre was that of the earth. This is the foundation on which the whole doctrine of the ſphere depends, and the application and computation of ſpherical triangles, which may be conſidered as the greateſt part of practical aſtronomy, and a deficiency in this renders the whole imperfect. Let the horizon of the place G, be repreſented by 2, G, 3, and it is the true one, although it be not that derived from obſervations, for it is a tangent to the circular meridian 4, G, 5; but, if the horizon C G T be taken for the horizon of the place G becauſe the direction of gravity is perpendicular thereto as G H N E, then the deviation is O N, and O is the true centre, but N the falſe one, and they differ from each other by the angle of deviation; and no ſpherical triangles can be formed of great circles equal in radii, whoſe centres are ſome at O, and others at N, ſo as to compare with each other as required in the doctrine of the ſphere.

405. The ſame truth may be deduced from other conſiderations. Let T, *b*, *a*, 1, repreſent an arch of the heavens, and O the centre of that arch. Then becauſe of the vaſt diſtance from O to T and *b*, the earth's ball at O will be but as a point, and the latitude of a place *b* on the earth will be in the line *b* O. To define a degree of latitude by the uſual method of the apparent alteration of a degree of zenith diſtance, is including in that degree of latitude the difference between the deviation of the direction of gravity at one place and at the other. But whilſt right lines may be drawn from all parts of the concave hemiſphere to one centre as to O, and thoſe radii are equal, the circumference of every circular plane, of an equal diameter with that of the concave hemiſphere and paſſing through the centre O, may be a baſe and horizon, on which ſpherical triangles may ſtand and be formed, and be ſo connected together one after another, as to circumſcribe the heavens if required; and all the calculations and obſervations relative to thoſe triangles will be ſtrictly conformable with the doctrine of the ſphere, but not otherwiſe. For if ſome of thoſe planes have O for the centre, and others N, others H or E; ſince the radii of all great circles of the ſphere are equal to each other, it would follow, that ſome of thoſe arches or ſides of the triangles would be coincident with the ſurface of the concave hemiſphere, others would fall without or

beyond

beyond it, and others ſhort of that ſurface, and others part beyond and part ſhort of that ſurface; all which, ariſing from a ſuppoſed eccentricity, is abſurd, and contrary to the plain principles of the ſphere, on which the doctrine of ſpherical triangles is formed. The inferences drawn from theſe principles, are as follows:

406. In the conſtruction of ſpherical triangles within the concave hemiſphere of the heavens, we entertain no idea of the earth's magnitude, but conſider it as a point, and that point as the centre of the heavens. This is the foundation on which the determination of not only the longitudes but the latitudes of places depends. Without this principle no meaſure of a degree of latitude can be known, becauſe at one part of that degree, the direction of gravity may be to one point in the polar axis, and at another part of the ſame degree of latitude towards ſome other point of the axis; and whilſt this deception is introduced, no meaſure of a degree of latitude, as intercepted between two radii, drawn from the earth's centre to the extremities of a degree of the meridian in the heavens can be obtained. And this only is a degree of latitude.

407. The longitude principles are of the ſame kind. Suppoſe a ſolar eclipſe to happen when the ſun is nearly on my meridian; to a perſon eaſt of me, it would be paſt noon; and to a perſon weſt of me ſhort of noon; but the time at all three places would be eſtimated from ſpherical triangles formed on the horizons of thoſe three places reſpectively; and the three hour angles being compared with each other would give the difference of longitudes of the places. In ſuch a caſe the three obſervers are conſidered as ſituated at one common centre, or, which is the ſame thing, they are ſuppoſedly ſituated on a ſpherical, not on a ſpheroïdal earth, and their horizons muſt be thoſe of a ſpherical earth, in order to which, as has been before proved, the obſerved latitude muſt be leſſened by the deviation of gravity. Finally, in ſuch ſpherical triangles as are formed on the concave hemiſphere of the heavens; we muſt either admit but one centre, that of the earth, to which all radii drawn from the concave hemiſphere are equal; or otherwiſe admit many centres; that formed by the equator and the poles being ſituated at the centre of the earth; but many others external to the earth, and deſtroying all

the

the harmony that ſubſiſts amongſt the circles of the ſphere. We proceed to examine what effects are hereby produced in ſeveral aſtronomical problems.

Problem.

408. What is the ſun's true amplitude of riſing, in the obſerved latitude 51° 31′ north, the longeſt day?

Calculation.

Sine declination —	° ′ 23 28	log.	9	6001181
Radius — —	90 0		10	0000000
		Sum	19	6001181
Coſine true latitude	51 12		9	7969930
Sine true amplitude	39 27		9	8031251
Without correction	39 47			
Difference, an error	20	north eaſtwardly.		

Another.

What is the ſun's true amplitude of riſing, in the obſerved latitude 51° 31′ north, the ſhorteſt day?

Calculation.

Sine declination —	° ′ 23 28	log.	9	6001181
Radius — —	90 0		10	0000000
		Sum	19	6001181
Coſine true latitude	51 12		9	7969930
Sine true amplitude	39 27		9	8031251
Without correction	39 47			
Difference, an error	20	ſouth eaſtwardly.		

Summer

	° ′
Summer error north westwardly	0 20
Winter error south eastwardly	0 20
Difference between them	0 40

PROBLEM.

409. Given P Z the colatitude 38° 29′ north,
P S the polar distance 66 32
Z S the coaltitude 30 25
Required the hour angle S P Z?

USUAL METHOD.

° ′		
38 29		
66 32		
30 25		
135 26		0 2060093
		0 0374924
67 43		9 9662920
37 18		9 7824643
		19 9922580
° ′ ″		
7 38 19		9 9961290
15 16 38		
	h ′ ″	
for	10 58 54	morning,
and	1 1 6	afternoon.

TRUE METHOD.

° ′		
38 48		
66 32		
30 25		
135 45		0 2030070
		0 0374924
67 52		9 9667562
37 28		9 7841177
		19 9913733
° ′ ″		
8 3 48		9 9956866
16 7 36		
	h ′ ″	
for	10 55 30	morning,
and	1 4 30	afternoon.

				h ′ ″
Time by the common calculation	—	—	—	10 58 54
by the true	—	—	—	10 55 30
Common calculation too soon	—	—	—	3 24

This is an observation made between the east or west and the north, in north latitude and north declination.

ANOTHER.

410. Given the colatitude 38° 29′ north, the polar distance 66° 32′, and the coaltitude 60° 9′.

Required

Required the hour angle?

Usual Method.		True Method.	
° ′		° ′	
38 29		38 48	
66 32		66 32	
60 9		60 9	
165 10	0 2060093	165 29	0 2030070
	0 0374924		0 0374924
82 35	9 9963513	82 44	9 9964977
22 26	9 5816177	22 36	9 5846651
	19 8214707		19 8216622
° ′ ″	9 9107353	° ′ ″	9 9108311
35 29 27		35 28 23	
70 58 54		70 56 46	
h ′ ″		h ′ ″	
for 7 16 5 morning,		for 7 16 13 morning,	
and 4 43 55 afternoon.		and 4 43 47 afternoon.	

	h ′ ″
Time by the common calculation —	7 16 5
by the true — — —	7 16 13
Common calculation too late — —	8

This is an observation made nearly east or west, and therefore but little difference between the usual method and the true method.

Another.

411. Given the colatitude 38° 29′ north, the polar distance 66° 32′, and the coaltitude 85° 15′.

Required the hour angle?

Usual Method.		True Method.	
° ′		° ′	
38 29		38 48	
66 32		66 32	
85 15		85 15	
190 16	0 2060093	190 35	0 2030070
	0 0374924		0 0374924

° ′			° ′		
95 8		9 9982546	95 17		9 9981510
9 53		9 2346249	10 3		9 2418141
		19 4763812			19 4804645
° ′ ″		9 7381906	° ′ ″		9 7402322
56 49 47			56 38 40		
113 39 34			113 17 20		
	h ′ ″			h ′ ″	
for	4 25 22	morning,	for	4 26 51	morning,
and	7 34 38	afternoon.	and	7 33 9	afternoon.

	h ′ ″
Time by the common calculation —	4 25 22
by the true — — —	4 26 51
Common calculation too late — —	1 29
	h ′ ″
Common calculation too ſoon by 1ſt Example	0 3 24
too late by 3d	0 1 29
whole difference	0 4 53

Another Example.

412. Given the latitude of the place by obſervation 51° 31′ north, the ſun's declination north 21° 9′, and the altitude cleared from refraction 5° 15′. Required the hour angle?

Usual Method.			True Method.		
° ′			° ′		
38 29			38 48		
68 51			68 51		
84 45			84 45		
192 5		0 2060093	192 24		0 2030070
		0 0002864			0 0302864
96 2		9 9975877	96 12		9 9999047
11 17		9 2915040	11 27		9 2977883
		19 5253874			19 5309864

° ′		° ′	
54 37	9 7626937	54 22	9 7654932
109 14		108 44	
h ′ ″		h ′ ″	
for 4 43 4 morning,		for 4 45 4 morning,	
and 7 16 56 afternoon.		and 7 14 56 afternoon.	

				h ′ ″
Time by the common calculation	—	—	—	4 43 4
by the true	—	—	—	4 45 4
Common calculation too late	—	—	—	2 0

In this observation the sun's bearing was between the east or west and the north, in north latitude and for north declination.

Another.

413. Given the colatitude and polar distance as before, but the altitude cleared from refraction 27° 15′. Required the hour angle?

Usual Method.		True Method.	
° ′		° ′	
38 29		38 48	
68 51		68 51	
62 45		62 45	
170 5	0 2060093	170 24	0 2030070
	0 0302864		0 0302864
85 2	9 9983663	85 12	9 9984742
22 18	9 5791616	22 27	9 5819236
	19 8138236		19 8136912
36 11	9 9069118	° ′ 36 12	9 9068456
72 22		72 24	
h ′ ″		h ′ ″	
for 7 10 32 morning,		for 7 10 24 morning,	
and 4 49 28 afternoon.		and 4 49 36 afternoon.	

Time

				h	′	″
Time by the common calculation			—	7	10	32
by the true	—	—	—	7	10	24
Common calculation too soon			—			8

In this observation the bearing of the sun was nearly due east or west, and therefore but little difference between the usual method and the true method.

ANOTHER.

414. Given the colatitude and polar distance as before, but the altitude cleared from refraction 47° 16′. Required the hour angle?

USUAL METHOD.			TRUE METHOD.		
°	′		°	′	
38	29		38	48	
68	51		68	51	
42	44		42	44	
150	4	0 2060093	150	23	0 2030070
		0 0302864			0 0302864
75	2	9 9850114	75	12	9 9853471
32	18	9 7278277	32	28	9 7298197
		19 9491348			19 9484602
°	′		°	′	
19	25	9 9745674	19	32½	9 9742301
38	50		39	5	

	h	′	″		h	′	″	
for	9	24	40	morning,	9	23	40	morning,
and	2	35	20	afternoon.	2	36	20	afternoon.

				h	′	″
Time by the common calculation			—	9	24	40
by the true	—	—	—	9	23	40
Common calculation too soon		—	—		1	0

		′	″
Common calculation too late by 1st observation		2	0
too soon by 2d		1	0
whole difference		3	0

PROBLEM.

415. Given PZ the colatitude 38° 29′ north,
SZ the coaltitude 30 25
SP the polar distance 66 32
Required the azimuth angle PSZ?

USUAL METHOD.		TRUE METHOD.	
38° 29′		38° 48′	
30 25		30 25	
66 32		66 32	
135 26	0 2060093	135 45	0 2030070
	0 2956053		0 2956053
67 43	9 9962920	67 52	9 9667562
1 11	8 3149536	1 20	8 3667769
	18 8128602		18 8321454
° ′		° ′	
75 14	9 4064301	74 53½	9 4160727
150 28 azimuth from the north.		149 47 azimuth from the north.	

By the usual method	—	—	150° 28′
By the true method	—	—	149 47
Usual method too great	—	—	41

Consequently, had the variation of the compass been taken by such an observation, it would have given that variation erroneous by two thirds of a degree.

ANOTHER.

416. Given the colatitude and polar distance as before, but the coaltitude 60° 9′. Required the azimuth?

USUAL METHOD.		TRUE METHOD.	
° ′		° ′	
38 29		38 48	
60 9		60 9	
66 32		66 32	
165 10	0 2060093	165 29	0 2030070
	0 0618149		0 0618149

° ′		° ′	
82 35	9 9963513	82 44	9 9964977
16 3	9 4416576	16 12	9 4455904
	19 7058331		19 7069100
° ′		° ′	
44 33	9 8529165	44 28	9 8534550
89 6 azimuth from the north.		88 56 azimuth from the north.	

By the usual method — — —	° ′ 89 6
By the true method — — —	88 56
Usual method too great — —	10

Had the variation been taken by this observation, the error would not have been so much as before, because the sun was nearer east or west.

ANOTHER.

417. Given the colatitude and polar distance as before, but the coaltitude 85° 15′. Required the azimuth?

USUAL METHOD.		TRUE METHOD.	
° ′		° ′	
38 29		38 48	
85 15		85 15	
66 32		66 32	
190 16	0 2060093	190 35	0 2030070
	0 0014942		0 0014942
95 8	9 9982546	95 17	9 9981510
28 36	9 6800560	28 45	9 6821349
	19 8858141		19 8847871
° ′		° ′	
28 44	9 9429070	28 52	9 9423935
57 28 azimuth from the north.		57 44 azimuth from the north.	

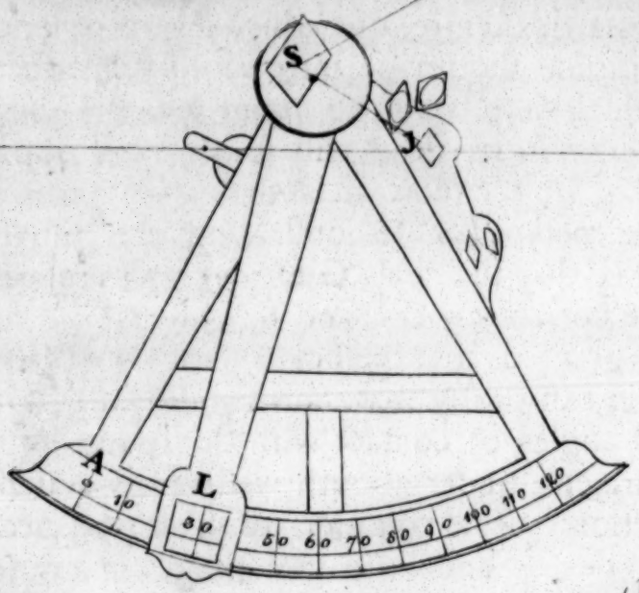
By the usual method — — —	° ′ 57 28
By the true method — — —	57 44
Usual method too little — — —	16

418. In theſe computations, by the true method, is meant, that when the obſerved latitude is leſſened by the deviation, the horizon then to be applied, is at right angles to a radius drawn to the earth's centre, after which ſeveral other corrections of the coaltitude ſhould be introduced to perfect the true method; which here being no room for, the uſual method is recommended in its ſtead, till ſome other opportunity of diſcuſſing this ſubject more at large.

419. The method of demonſtrating the uſual practice, is, by proving that the lines G H and 7 O are parallel, and therefore the horizons 3 G C and 6, 7, 8 are the ſame. This is taking for granted, that the horizons of the oblate ſpheroid, under a ſouth eaſtwardly and north weſtwardly direction, do make equal angles of contact with the ſpheroid. And the like for horizons under a ſouth weſtwardly and north eaſtwardly direction. The common ſwell of the ocean, which cauſeth the tides, is known to raiſe the ſea many feet higher at one place than another; and herefrom the dip of horizon may be affected ſometimes conſiderably. Was it but 5′ of a degree under one direction, under a contrary direction, the error might be doubled. And it may be farther queſtioned, how conſtantly the refraction near the horizon continues under the various agitations of the horizontal vapours?

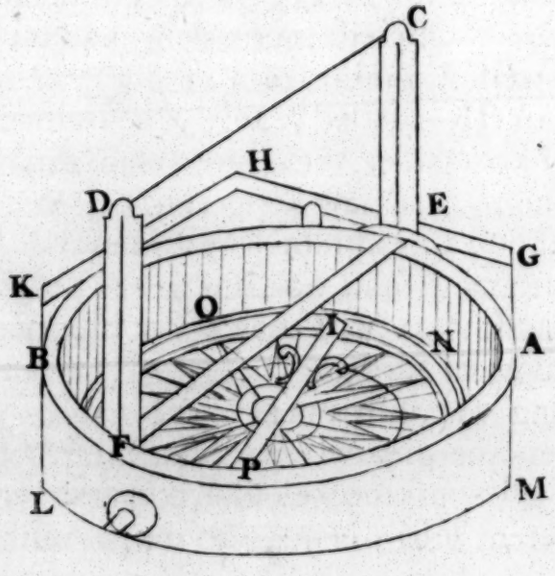
420. I have an excellent braſs Hadley's ſextant of a foot radius, which was divided by continual biſection after the manner deſcribed at article 226 of this work. By this inſtrument I have taken the latitude in London, ſome hundreds of times, and never differed more than half a minute of a degree; yet all were a minute of a degree too little. The inſtrument is correctly divided; and, by ſome other experiments which I have made, the ſame quadrant, by help of taking the medium of a few obſervations, has determined the latitude without an error of 5″ of a degree. With the ſame inſtrument, and a fluid horizon as perfectly at reſt and level as it was poſſible to be, I took other meridian altitudes, which differed ſo as to ſurprize me, as the greateſt care was taken in adjuſting the inſtrument, and making the obſervations. If when the greateſt care and diligence have been exerted, the altitudes of the celeſtial bodies have come out different by different inſtruments, the times muſt neceſſarily have

Hadley's Quadrant & Sextant.



This is the usual Form reversed, and seems best suited for the Eye & Hand.

The New Azimuth Compass.



have come out different, by the principles which have been before delivered. And the latitude, as found from two altitudes and the elapſed time, is affected by the ſame cauſe; for, if the elapſed time between two obſervations made on the ſame day, be kept ever ſo truly by a time-keeper, it appears that the difference between the two hour angles, as inferred from a computation of the time, having the obſerved latitude, the obſerved altitude, and the declination, may differ from each other ſeveral minutes of time; and therefore, as it is certain that no time-keeper can keep time too truly, to be applied in this method of finding the latitude and the time at the ſhip, by two altitudes and the elapſed time; it follows, that this correction, for the deviation of gravity, ought to be applied in that problem, as particularly as in others.

421. It has been ſhewn how the geographical longitudes of places on land may be determined from aſtronomical obſervations made at thoſe places; we proceed to ſhew how the longitudes of other places which are either contiguous to, or remote from ſuch places, may be aſcertained by obſervations made on the land. And here it will be requiſite for the practical aſtronomer to apply the doctrine of plane triangles.

422. The places, whoſe latitudes and longitudes have been ſo aſcertained from aſtronomical obſervations, may be conſidered as one or other of the following kinds; firſt, as a city or large town; ſecondly, a fortified place; thirdly, an harbour; fourthly, an eminence, headland or cape; fifthly, a village, houſe or place in the interior part of a country, or otherwiſe near the ſea ſhore. From all or any of which places, the latitudes and longitudes of other contiguous and remote places may be inferred and carried on to no inconſiderable diſtances, by the principles of plane and horizontal triangles; in the practice of which, the Hadley's quadrant, or, what will be better, the ſextant, and the new azimuth compaſs will likewiſe be here uſeful; particularly, when the variation of the needle is nearly known. In the uſe of the quadrant or ſextant, ſuppoſing O and B two objects at any conſiderable diſtance from each other, and alſo from the obſerver; the object O is brought by reflection from the great mirror S, to conicide by direct viſion, with the object B, in which caſe, the arch A L ſhews the angle made

made by the two right lines drawn from the obſerver, to O and B. When the diſtance is ſmall, this method is not correct, becauſe the places S and I, one being on the great mirror, and the other on the little mirror, are not coincident with each other. This produceth a parallax in near diſtances, but no perceptible error in great ones; and what the error is in near diſtances, is beſt determined by experiments. In the new azimuth compaſs, I P repreſents the magnetic north and ſouth, and F E or D C the bearing of the ſun or any other object at the time of obſervation; the card being ſunk at a proper diſtance below the ſurface of the box, to make room in a perpendicular direction on the ſide of the box within, for the diviſions and ſubdiviſioos of the degrees; by which method of graduating, the poſition of the needle is the better known at ſea, whilſt it is agitated by the motion of the ſhip.

423. But to ſhew the method of correcting the geographical ſituation of places in maps, and alſo the correction of the charts of the coaſts, by aſtronomical obſervations; ſuppoſe, for inſtance, the imperial city of Frankfort, in Germany, was one of thoſe places, and its latitude is determined 50° 6′ 0″ north, and longitude 0^h 34′ 18″ eaſt from Greenwich. Suppoſe Cologne, another place in Germany, whoſe latitude is determined 50° 55′ 0″ north, and longitude 0^h 28′ 18″ eaſt of Greenwich. Again, ſuppoſe Gottingen, another place in Germany, whoſe latitude is determined 51° 32′ north, and longitude 0^h 39′ 34 eaſt from Greenwich: then we have by aſtronomical obſervations as follows, between Gottingen and Frankfort, or Cologne:

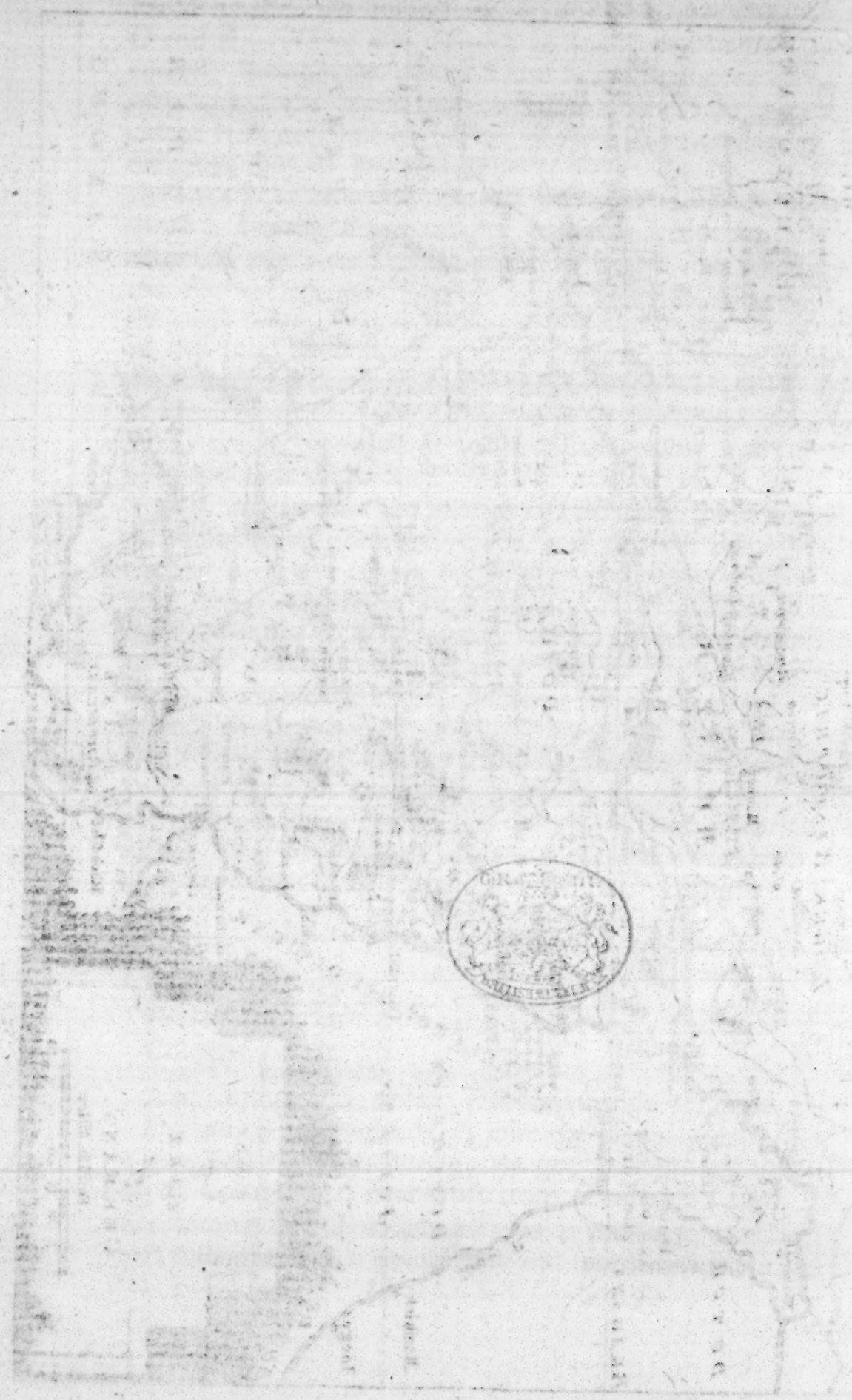
	Latitude	Longitude	Diff. Lat.	Diff. Long.
	° ′	h ′ ″	° ′	h ′ ″
Gottingen	51 32	0 39 34		
Frankfort	50 6	0 34 18	1 26	0 5 16
Cologne	50 55	0 28 18	0 37	0 11 16

And ſaying, As coſine middle latitude : radius : : Difference of longitude in minutes of a degree : diſtance in geographical miles nearly; we get the ſouthing and weſting of Cologne and Frankfort from Gottingen; thus.

	Southing	Weſting
Cologne	37 miles	105,80 miles
Frankfort	86	49,90

And

A Map
of
PART
of
GERMANY.
British Miles
5 10 15 20 40
ZUTPHEN
LOWER BISHOPRIC
DUTCH
DUTCHY
Nimmegen
Cleve
Emmerick
Rees
Steinberg
Wesel
Burick
Dutchy of Geldres
BRABANT
Rheinberg
Wachtendonk
Meuse R.
Maestrich
DUTCHY OF JULIERS
Liege
Crevelt
Head-quarters of Marshal de Contades
Duysburg
Duffeldorp
Nuys
COLOGNE
ELECTOR. OF COLOGNE
DUTCHY OF BERG
Bonn
Andernach
Coblentz
Moselle R.
Rhine R.
Mayence
Steinhorst
Stadtloon
Coesfeld
Lette
Vehlen
Dulmen
Munster
Head-quarters of Prince Ferdinand
Telgte
Wolbeck
Warendorp
Osthen
Alen
Rittberg
Lippe R.
Recklinghausen
Ham
Soest
Lipstadt
Emser R.
Essen
Dortmund
Unna
Neun
Ruden
Arensberg
COUNTY OF MARCK
Roer R.
Xanten
Balve
Meschede
Weerdet
Ludenscheidt
Lutheim
Duyts
Landsberg
Ouart
Siegberg
Wildenberg
Siegen
Blanckenberg
Kirpel
Seyn
Serentz
Hermanstein
Runckel
Nassau
Dietz
County
HESSE RHEINFELS
Rheinfels
Wisbaden
Hoyerdan
Greven
Ravensberg
Bielefeld
PRINCIPALITY OF MINDEN
Horn
Steinheim
BISHOPRIC OF PADERBORN
Paderborn
Lichtenau
Rintelen
Pyrmont
Minden
Oldenburg
Hannover
Hildesheim
Bishopric to Cologne
Hamelen
Holl
Oldendorp
Bevern
Dassel
Einbeck
ELECTORATE OF HANOVER
Bracket
Corbach
WALDECK
Wildeck
CASSEL
Arolgen
Westphalen
Carlshaven
Northeim
Gottingen
Dudenhault
Munden
Sauterhausen
Allendorf
Lichtenau
Eschwege
Treffurt
Eisenach
Fritzlar
Homburg
Fulda R.
Alirsfeld
Ziegenhayn
Lauterbach
FULDE
PART OF UPPER SAXONY
Smallenberg
Hatzfeld
Heyna
Dillenburg
Marpurg
Wetzlar
Gieffen to Darmstad
Wilburg
Ortenberg
Budingen
Fridburg
Hanau
COUNTY OF HANAU
Frankfort
Head-quarters of Prince de Soubise



And reducing these distances to English miles, they become nearly thus,

	Southing	Westing
Cologne	43,2 English miles	123,5 English miles
Frankfort	100,3	58,3.

Which. being compared with the map, gives for

	Southing	Westing
Cologne	39 English miles	132 English miles
Frankfort	97	68.

By which differences, the errors to which geographers are frequently subject will appear to be often very considerable. The like comparisons might have been made concerning Hanover, or any other places in the map; and the distances of those places, from one another respectively, must of course become erroneous by the map, as their differences of latitude and easting or westing differ from the numbers found by astronomical observations.

424. Thus, if the distance from Gottingen to Frankfort be required, we have the difference of latitude 100,3 English miles, and the westing 58,3, English miles, from which the distance is 113,4 English miles; this, by the map, is 120 English miles. Again, if the distance from Gottingen to Cologne be required; we have the difference of latitude 43,2 English miles, and the westing 123,5 English miles, from which the distance is 130,8 English miles; this, by the map, is 136 English miles. And if the distance from Frankfort to Cologne be required; the difference of latitude is 65,2 English miles, and the westing 57,1 English miles, from which the distance is 86,3 English miles; this, by the map, is 85 English miles. And farther, comparing these respective distances with each other, they will be in miles, from

	By Map	By Observ.	Differ.
Gottingen to Frankfort	120	113,4	6,6
Gottingen to Cologne	136	130,8	5,2
Frankfort to Cologne	85	86,3	1,3

In this comparison, it may be observed, that because the distance between Frankfort and Cologne is but small, there-

fore the error is leſs than that of the other diſtances; ſuch correctneſs is often owing to the ſtraightneſs of the roads, or eaſy acceſs to proper ſtationary objects. I have not made the computations here by the moſt correct method, this being ſufficient for uſual caſes; otherwiſe, the ſpherical triangles, for greater diſtances, ſhould be applied.

425. If ſuch errors are to be met with, in countries where arts and ſcience furniſh every aſſiſtance for correcting the geographical ſituation of places; when the places are at no greater diſtances from each other, and where the beſt geographers and aſtronomers are not wanting; what are we to expect from the delineation of places which are at much greater diſtances from each other, and where aſtronomical obſervations have not been made for the longitude, for intermediate ſpaces of ſome hundreds of miles? and ſuch are many places near the coaſts which ſeparate the lands from the oceans, in countries which have been but little viſited; and the like may be obſerved concerning rivers of great length running between the lands. The only remedy againſt theſe errors is the practice of aſtronomy, whereby the proper ſtationary points may be well aſcertained, and therefrom the latitudes and longitudes of other intermediate places, until the whole is filled and made as perfect as poſſible.

426. But in the execution of ſuch a work, the artiſt will want more than ordinary helps and aſſiſtance; the mere drawing by eye, will exhibit but an imperfect outline of the boundary which is wanted to be accurately delineated. The variety of objects and obſtructions he may meet with in tracing out the line that bounds or limits ſuch a tract, will ſubject him, every hour, to errors which may become at laſt very great, and thereby render the whole performance of but little uſe. And therefore, any method whereby theſe difficulties may be overcome, and truth and certainty attained in topographical deſcriptions, cannot but be of public utility. The aſtronomical obſervations are the baſe and foundation on which all his work ſtands; and an error in theſe, or for want of them, his beſt performance will be little better than a picture coloured and ſhaded with the moſt exquiſite elegance and judgement of colour and ſhading only; but having its outlines erroneous, exhibits the utmoſt imperfection and deformity. An extenſive ſurvey having

having its outlines or boundary ſtations well fixed, has its principal part finiſhed; but without this, the whole is irregularity and error.

427. The ingenious Dominico Caſſini, by birth an Italian, but for his aſtronomical abilities engaged as principal aſtronomer at the Royal Obſervatory at Paris, and cotemporary with Flamſtead and Sir Iſaac Newton, may be reckoned amongſt thoſe of the firſt claſs, who have contributed towards the correction of geography by help of aſtronomical obſervations. It is recorded that, for more than eighty years ago, this quickſighted aſtronomer had been travelling through Italy, Germany, France, England, and other places; and wherever he came, every opportunity was embraced by him, in making obſervations for the latitudes of places, by the ſun's meridian altitude, and the meridian altitude of the pole ſtar above and below the pole; and in making obſervations of the immerſions and emerſions of the ſatellites of Jupiter; whereby the maps and charts became almoſt immediately corrected in longitude as well as latitude, of places where he obſerved, ſo as to leave but little room for any amendment.

428. From thoſe obſervations, joined with topographical deſcriptions, this aſtronomer and his aſſiſtants are ſaid to have drawn the coaſt of the weſtermoſt part of France; and ſo correctly, as to admit of but little (if any) improvement; except ſuch additions as may have been introduced by buildings of towns, villages, &c. near the coaſt; and ſuch ſmall changes and alterations as have ariſen amongſt the ſands, by the waſhing and motion of the ſea. And, as this was no imperfect pattern for a coaſt ſurvey, we proceed to ſhew how it may be ſuppoſed to have been taken; and how any other, under like difficult circumſtances, may be accurately delineated.

429. In this ſurvey the firſt things to have been determined, were the latitude and longitude of Breſt. The aſtronomical obſervations gave the latitude 48° 23′ 0″ north, and the longitude 0^h 27′ 23″ weſt from Paris; theſe two obſervations effectually ſettle the diſtance and poſition of Breſt from Paris to the greateſt exactneſs that can be required. The next ſtep to be taken, was to ſurvey the adjacent coaſt, in the performance of which, the aſſiſtance of plane tri-

angles and drawing by the eye, could not but be of the greateſt utility. Here I ſhall introduce a capital improvement in the practice of ſurveying, by a proviſion of proper lines in the form of a rete or net-work, whereby to inſert the eaſting and weſting, northing and ſouthing, as they may be found by obſervation; from whence the diſtance may be inferred, not much unlike the method of delineating the diſtance, difference of latitude and departure, in navigation. And ſuppoſing the execution of ſuch a ſurvey to be again performed, it may be effected as follows.

430. Having ſettled the place of Breſt, and provided a delineation of ſquares, or in other words of parallel and perpendicular right lines drawn equidiſtant from each other, and at ſuch diſtances as are ſuitable for the ſurvey, as in the delineation annexed; the ſurvey may be begun, either on the ſea or on the land. Firſt, it ſhall be ſhewn how it is to be conducted on the ſea; and ſecondly, on the land.

431. Let *a* and *b* repreſent the places of two veſſels on the ſea, whoſe diſtance is meaſured as correctly as the practice of meaſuring on the water by the log line, or any other method will admit; and from thoſe two ſtations, let the angles 13, *a*, *b*; *c*, *a*, *b*; 5, *a*, *b*; 2, *a*, *b*; &c. likewiſe, the angles 13, *b*, *a*; *c*, *b*, *a*; 5, *b*, *a*; 2, *b*, *a*; &c. be taken; from which, by trigonometry, the diſtances to the coaſt, and points, 13, *c*, 5, 2, &c. are determined. And drawing the line of the coaſt, according to art, through thoſe points, and inſerting the other particulars of villages, caſtles, churches, houſes, capes, rocks, breakers, ſands, &c. the eye-drawing is ſketched, and fit to be farther finiſhed, or introduced into the general plan, map, or chart.

432. In the performance of the latter part of this work, the ſurveyor will meet with innumerable difficulties, tending (as hath been before obſerved) to introduce error in his performance; but ſuch may be got over, in a very great meaſure, by an application of the experiments before delivered in this work, concerning the expanſion or contraction of paper, by paſſing through the rolling-preſs in copper-plate printing.

434. By the foregoing experiments, it appears that when a ſquare copper-plate print of ſquares, or chequered lines, has been paſted properly on a ſmooth board, its equality of dimenſions in length and breadth may be very nearly retained; and

PART OF BRETAGNE

BAY OF DOVARNENEZ

POLDAVIDS BAY

PASSAGE DU RAS

Miles

Conquet White Bay

Conquet Road

BREST

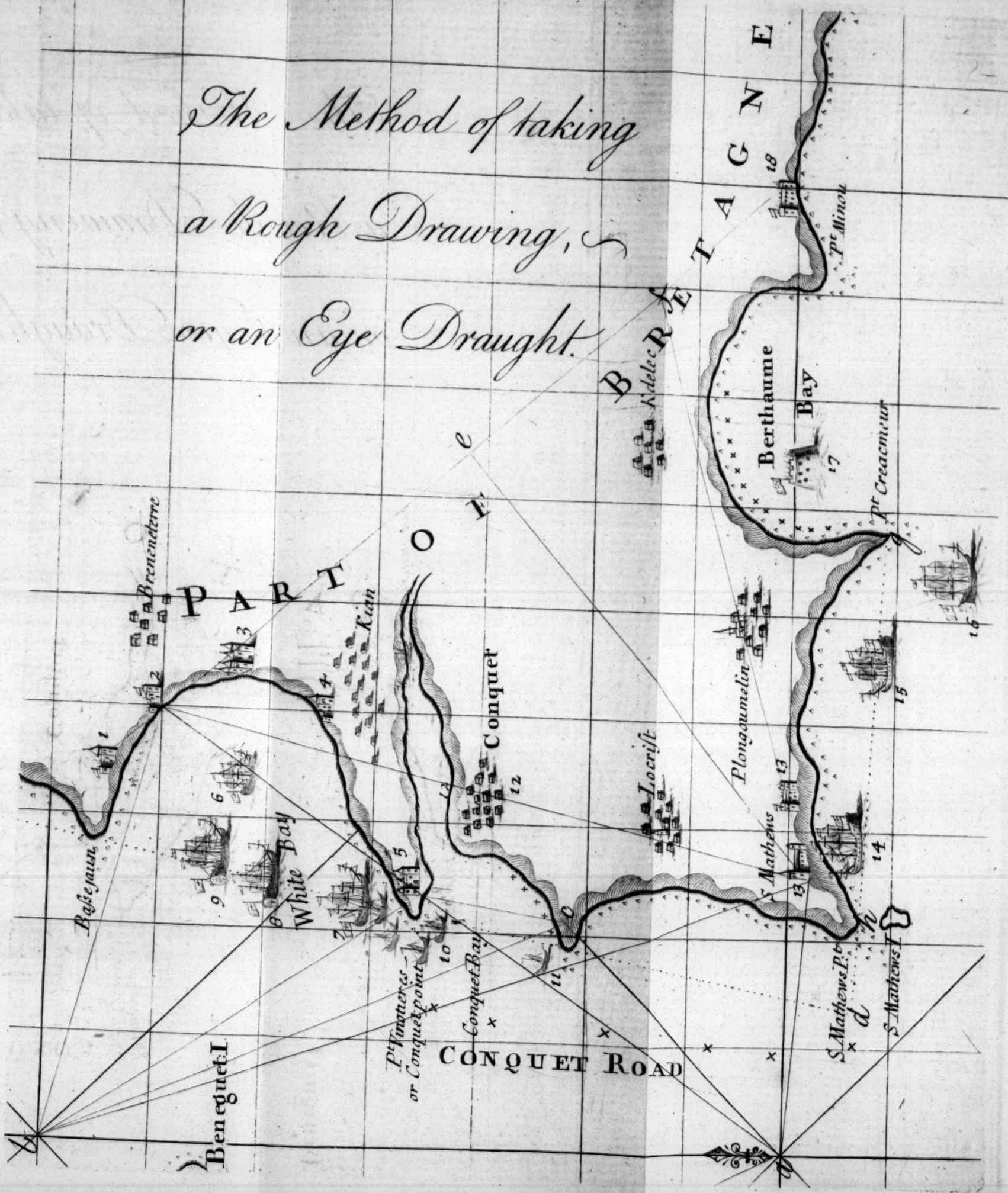
Remarks

Sands which Cover and Uncouer
Banks always Covered
Ouse which Covers and Uncouers
‡ Rocks which never Uncouer
+ Rocks which Uncover after Spring Tydes
Rocks always Uncouered
Rocks which Cover and Uncouer
Lines which Shew the Dangers
The Numbers shew the Depth of Water in Fathams.

Runegon
Banc Pourceaux
Basse jaune
Brneterre
Sable Poulmeguer
Point Ilien
I Quemignes
I Litiri
L Morgat
Bareſablon
P Vinotiere
Vintiere
Roch Noire
I Beneguet
Conquet
N du Bilou
S Mathieu
Ville Neuve
Kejan
Plongomelin
Kodelec
Road Berthaume
Batterie de Leon
Plouzane
M Colbroc
le Gouldt
Anglum
Gondro
Fillettes
Cormorandiere
Roscanuel
Remart
P Camaret
Lieval de Marque
Isle et Pt
Treberon
Road Camaret
M Camaret
3 Car Lanconet
Fret
Lanauaux
Poulmie
M Poulmie
Rigonou
Camaret
P Tavelle
Crodon
P Rulariet
P Telberton
S Laurens
I Lavor
P Besaguer
Morgard
P Morgard
le Taureau
P Profunde
le Verres
I Gueneran
le Boue
P Tengvera
S Nicolas
Pierre Longue
P Loye
Bec de Cheure
Basse Vieille
Basse du Lis
le Cherre
le Cherreau
Cha Dinant
Dinant
le Menan
la Vandree
Ile Goemont
la Parquet
Les Trepiedes
Roche Plate
P Perceaux
Perceaux
Lozzdonnen don
R Corbin
Ile Vierge
Le Forhol
Modenou
Rodinac
Rodic Kentonec
Cornabrador
Cornoctreac
Harpuen
Hernastu
Roquedon
Carnacantreas
Basses Tas
Pt Bemel
Trepiers
Rochat Noires
Knaon
Knout
Kynard
Cledon
P Duellen
la Paneu
le Forbic
le Rastic
P Talahan
P de Loedet
P de Lezcom
Goulien
P de Lezme
Isles Vedell
P du chaine
Burzec
P Brem
Isle et Pt
P Meglois
P Bazurgleus
Pt la Iliment
Polan
le Binoche
la Jument
P de Labou
P R Blanche
Als
Pt Bemel

BREST
BRETAGNE
BAY OF DOVARNENEZ
POLDAVIDS BAY
M. CASSINI'S SURVEY.

The Method of taking a Rough Drawing, or an Eye Draught.



and therefore theſe experiments point out a remedy againſt one of the greateſt evils that attends the uſual practice of ſurveying in general, and of the coaſts in particular, which may be applied as follows.

434. Let a proper number of thoſe chequered prints or ſquares be paſted, one by one, each on a ſquare board, ſo that a certain number of thoſe boards with the lines on them, being joined, may be ſufficient to contain the plan or map that is to be made. Theſe being joined to one another, it will eaſily follow that all the principal lines paſſing to great diſtances over theſe boards, on which the ſurvey is intended to be drawn, may be drawn on them, almoſt as correctly as on one large flat board, and hereby the capital ſtationary points will be found on the plan very nearly as correctly as by any method whatſoever. But the peculiar advantage attending this method, ariſeth from the ſeparation of the parts; for any one of theſe parts may be taken to the place of the ſurvey, and drawn on upon the ſpot, without any errors ariſing by expanſion or contraction of the paper, as happens in the uſual practice. And ſo the whole may be filled, and the ſurvey finiſhed; after which it is eaſy to make proper copies, reductions, or embelliſhments, as ſhall be thought fit.

435. In coaſt ſurveying, a baſe line may ſometimes be made from one ſtationary point to another on the land. Thus, if from 1 to *f*, or to 18, be made a baſe line, and that line meaſurable; if the angle at *e* be 90°, and the diſtance *e f*, alſo the angle at *f* taken, the diſtance *e*, 13, is determined; in like manner may 18, 13, be determined; and the diſtances of any other places.

436. On the ſea, the ſituation of capes and headlands may ſometimes be determined by their coinciding in the ſame direction. So, if the point *d* be determined on the plan, and the headlands at *c*, 5, and 1, are in one and the ſame direction as ſeen from *d*; the diſtances *d*, *c*; *c*, 5; and 5, 1, will determine thoſe places on the plan. The like may be ſaid of the places, *g* and 18, or any others. And thus may the whole be carried on and finiſhed.

437. Otherwiſe, the ſurvey may be carried on by means of plane triangles formed nearly coincident with the horizon; and this method may be extended to great diſtances, when an

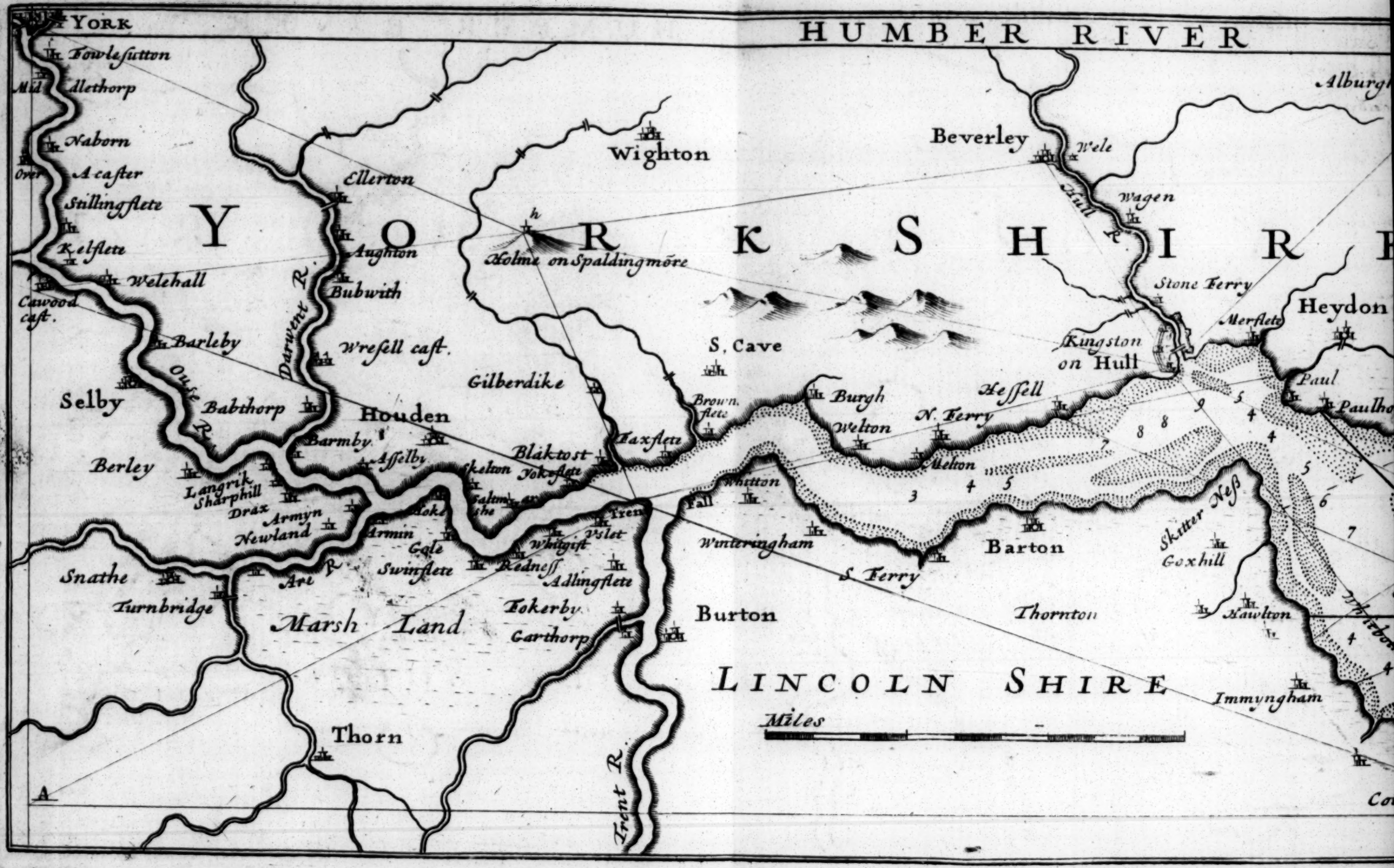
an allowance is made for the curvatureof the earth's ſurface, whereby it differs from a plane. An inſtance may be given in the ſurvey of a river, and part of a coaſt.

438. The latitude of York by obſervation is 53° 28½′ north, and its longitude from London 0ʰ 4′ 5″ weſt. Let A be the latitude of Grimſby as derived from aſtronomical obſervation, which ſuppoſe to be 53° 44½′ north, and its difference of longitude may be inferred from the difference of latitude between York and Cawood, and between Cawood and A; the angle made by the line drawn from Grimſby to Cawood, and from Cawood to A, being ſuppoſedly known. Here the meridian of York drawn to A, and the eaſting from A to Grimſby, will be baſes, on which triangles may ſtand, and all the diſtances from York, with the poſitions may be found. But it is neceſſary that either the line drawn from York to Cawood, or from Cawood to A, be known, becauſe then, theſe may be made baſes wherefrom the point *b* near Trent may be found. And by this method of proceeding, the ſurvey may be carried on to Hull, to Alborough, to Spurn-head, &c. and the line of the coaſt determined.

439. Otherwiſe, the ſurvey might have begun at B on the ſea; and, knowing the ſituation of the point B, or rather its diſtance from Aldborough and Spurn-head, by help of the intercepted angles, the coaſt might have been delineated. And then, the horizontal triangles being continued to York, or to any other place whoſe latitude and longitude was known, the latitudes and longitudes of places on the coaſt might have been determined.

440. The latitude of Dublin, in Ireland, is 53° 20′ north, and its longitude from London 0ʰ 24′ 30″ weſt. The latitude of Cape Clear, near Ireland, is 51° 18′ north, and its longitude from London 0ʰ 44′ 40 weſt. Whence the diſtance from Dublin to Cape Clear may be found, after the ſame manner as the diſtances of places in Germany were found by the foregoing calculations; and thereby the coaſt between Cape Clear and Dublin the more accurately delineated with the intermediate places. And by the ſame method may the ſituation, latitudes, and longitudes of other places in Ireland be determined.

441. The latitude of Edinburgh. in Scotland, is 55° 58′ north, and its longitude 0ʰ 12′ 51″ weſt of London. By aſtrono-


YORK
HUMBER RIVER
Fowlesutton
Allethorp
Naborn
Acaster
Stillingflete
Kelfleete
Welehall
Barleby
Selby
Babthorp
Houden
Barmby
Asselby
Berley
Langrik
Sharphill
Drax
Armyn
Newland
Snathe
Turnbridge
Thorn
Ellerton
Aughton
Bubwith
Wreſell caſt.
Darwent R.
Ouſe R.
Ouſe
Wighton
Holme on Spaldingmore
Gilberdike
Blaktoft
Yokefleet
Swinflete
Redneſſ
Adlingfleete
Tokerby
Garthorp
Marsh Land
Ari R.
Trent R.
S. Cave
Brown. fleet
Burgh
Welton
Melton
Whitton
Faſl
Winteringham
Burton
Beverley
Wele
Hull R.
Wagen
Stone Ferry
Kingston on Hull
Merſleet
Heydon
Paul
Hesſell
N. Ferry
Barton
Skatter Neſs
Goxhill
Thornton
Hawlton
Immyngham
LINCOLN SHIRE
Miles

HUMBER RIVER

Alburgh

Beverley

Wele

Hull R.

Wagen

Grimstongarth

K S H I R E

Stone Ferry

Heydon

Holder Neß

Waxham

Merflete

Kingston on Hull

S. Cave

Paul

Paulhelm

Brown. flete

Burgh

Welton

Hessell

N. Ferry

Patrington

Humpton

Melton

Whitton

Hall

Skitter Neß

Newton

Dimmilton

Wintringham

Barton

S. Ferry

Goxhill

Burstallgarth

Kelnsey

Sunk

Sand

Burton

Thornton

Hawlton

Flamborough

Den I.

Hawk

Road

Spurn Head

LINCOLN SHIRE

Immyngham

Miles

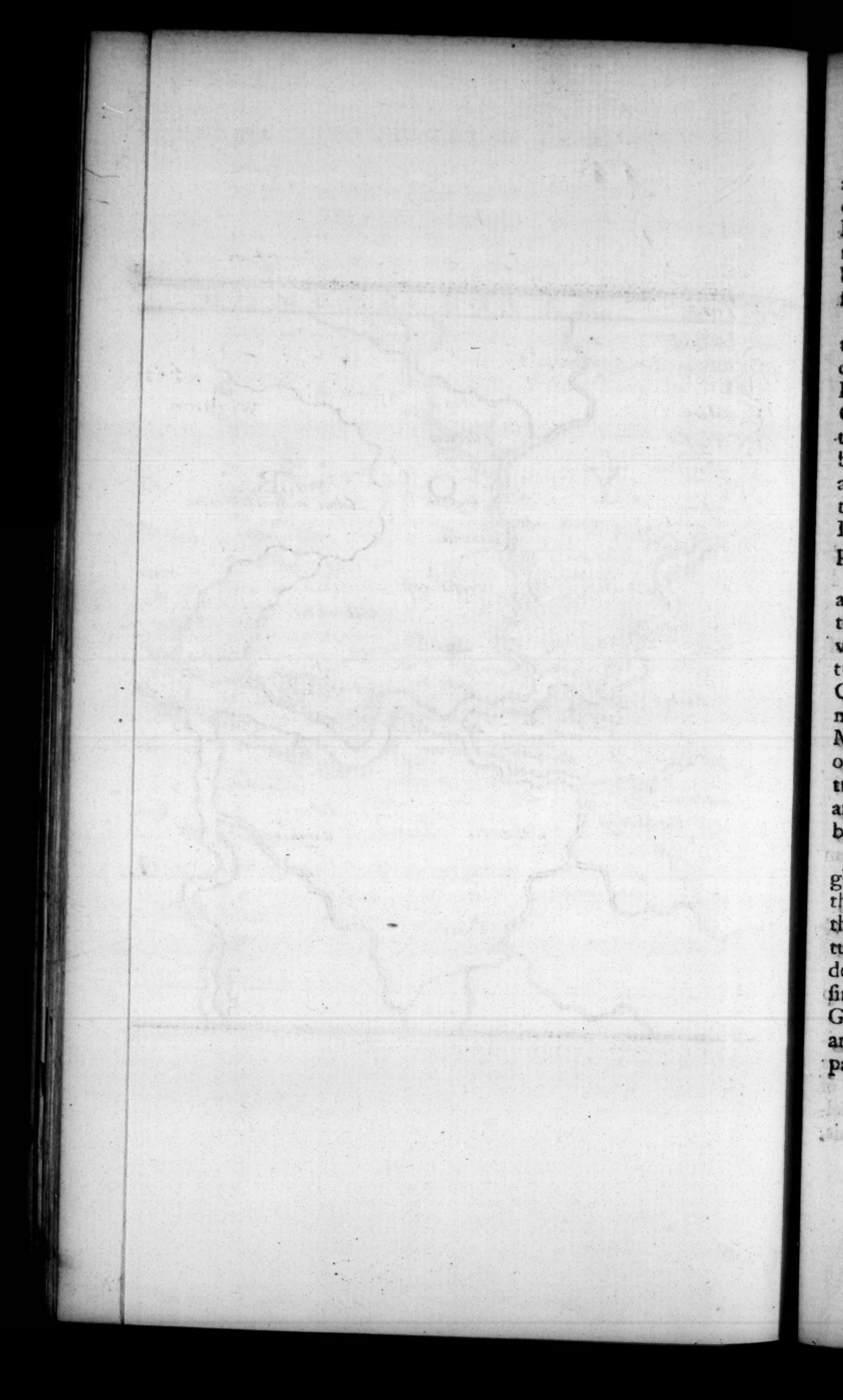
Clee Neß

Cotes

Grimsby

Clee

Rock



aſtronomical obſervations, the meridian of the ſouth point of Shetland Iſle, being nearly the ſame as the meridian of Edinburgh, the difference of latitude forms a baſe for correcting the north eaſt coaſt of Scotland. And the like may be underſtood for a part of any other meridian, and the ſituation of places derived from them.

442. As the ſituation of Breſt was determined, ſo have the ſituations of other places near the Britiſh channel been determined from aſtronomical obſervations. Thoſe near the French coaſt have been Calais, Boulogne, Diepe, Rouen, Caen, Cherbourg, St. Malo, and other places; from which the latitudes and longitudes of the intermediate places may be had without any material error, by the better ſort of charts and maps. On the Engliſh coaſt, ſeveral places have had their latitudes and longitudes correctly obſerved between Dover and the Lizard; and the intermediate places are pretty nearly known by the county ſurveys.

443. From Breſt along the coaſt of the Bay of Biſcay, as at Nantes, Rochell, Bourdeaux, and Bayonne, the latitudes and longitudes are known from aſtronomical obſervations. And on ſome parts of the coaſt of Spain and Portugal, as at Bilboa, Ferrol, Cape Finiſter, Oporto, Liſbon, Cadiz, and Gibraltar. And there are many places on the north coaſt of the Mediterranean, and in the iſlands of the Mediterranean, from the Straits of Gibraltar to the Iſland of Cyprus, and the coaſts of Syria and Egypt; whoſe latitudes have been determined from aſtronomical obſervations; and from which the ſituation of the intermediate places may be nearly known.

444. Towards the ſouth and eaſt parts of the earth's globe, the Iſland of St. Helena, the Cape of Good Hope, the French iſlands in the Indian ocean, ſeveral places on the coaſts of Malabar and Coromandel, have had their latitudes and longitudes accurately determined. And from a delineation of part of India, which I made ſome few years ſince, the latitudes and longitudes of places along the river Ganges and in the kingdom of Bengal may be nearly known; and the latitudes and longitudes of certain places in theſe parts, have been reckoned as follows:

Longitude

Longitude from Greenwich.

	Latitude		Longitude	
	°	′	°	′
Agra	27	8	78	16½
Scanderab.	26	32	77	19½
Gualour	26	11	77	49
Narvar	25	42	77	39
Seronge	24	22	77	8

These places in the road from Agra towards Brampur, &c.

Again,	°	″	°	′
Bindes	26	27½	78	41
Coulpe	26	1½	79	19
Antava	25	30	80	32
Eliabad	25	24	81	22½
Kenauge	25	21	81	33
Banares	25	20	82	45

These places in the road from Agra towards Banares.

Again,	°	′	°	′
Ferozibad	26	56½	79	30½
Ellagur	26	41	79	56
Sikindra	26	18½	79	29½
Corigenab.	26	6	80	1

These places in the road from Agra, by Corigenabad towards Eliabad.

Again,	°	′	°	′
Kanogy	26	41½	80	0
Ebrahinab.	26	46½	79	37
Ferogabad	27	2	79	33
Keif	24	30	80	39

These places in the road from Corigenabad northwardly.

Again,	°	′	°	′
Caramuſhp.	25	46	80	59
Lacknour	26	44	80	46
Oude	26	27	81	44

These in the road northwardly from Eliabad.

Again,	°	′	°	′
Bateah	26	50	84	8
Salempur	26	31	84	25
Jurki	26	30½	84	59½
Marian	26	55	85	1½
Siſarloor	25	56	83	57
Buxar	25	27½	83	58

These in the province of Bateah.

Again,	°	′	°	′
Mobilipur	25	14½	84	42
Arrover	25	8	84	39
Daudnagur	25	1	84	19
Gotaeli	24	51	84	6
Ecbarpur	24	12½	83	49
Rotaſgur	24	3	83	45

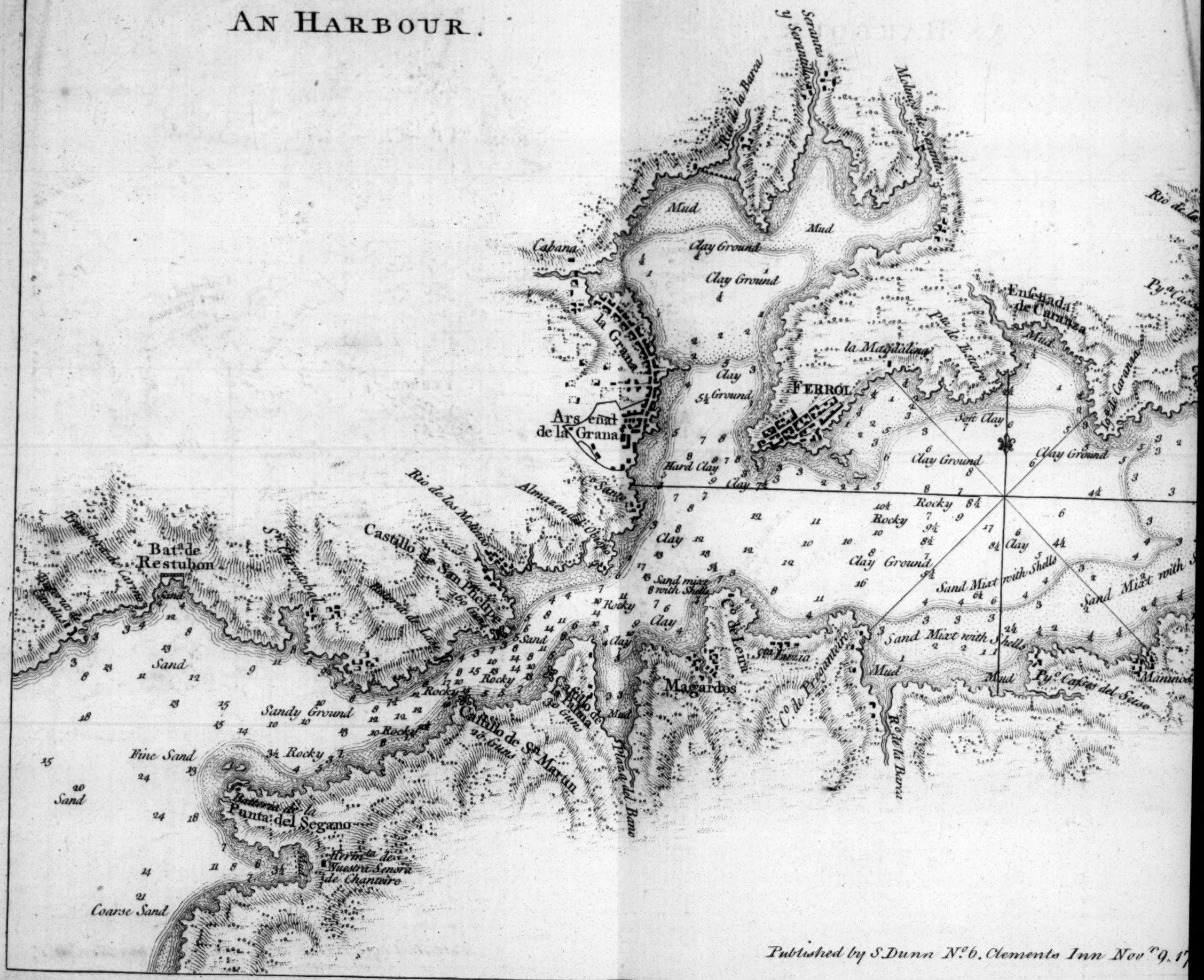
These places on the Soan river, and near it.

Again,	°	′	°	′
Saſeram	24	56	83	54½
Samelpur	21	52½	83	47

But it is probable that theſe may want correction by help of aſtronomical obſervations.

445. On the American continent, the latitudes and longitudes of many places are known near the coaſt that borders on the Atlantic ocean, from Newfoundland to the mouth of the Miſſiſippi river; theſe are Boſton, New York, Philadelphia,

The Method of Delineating
An Harbour.



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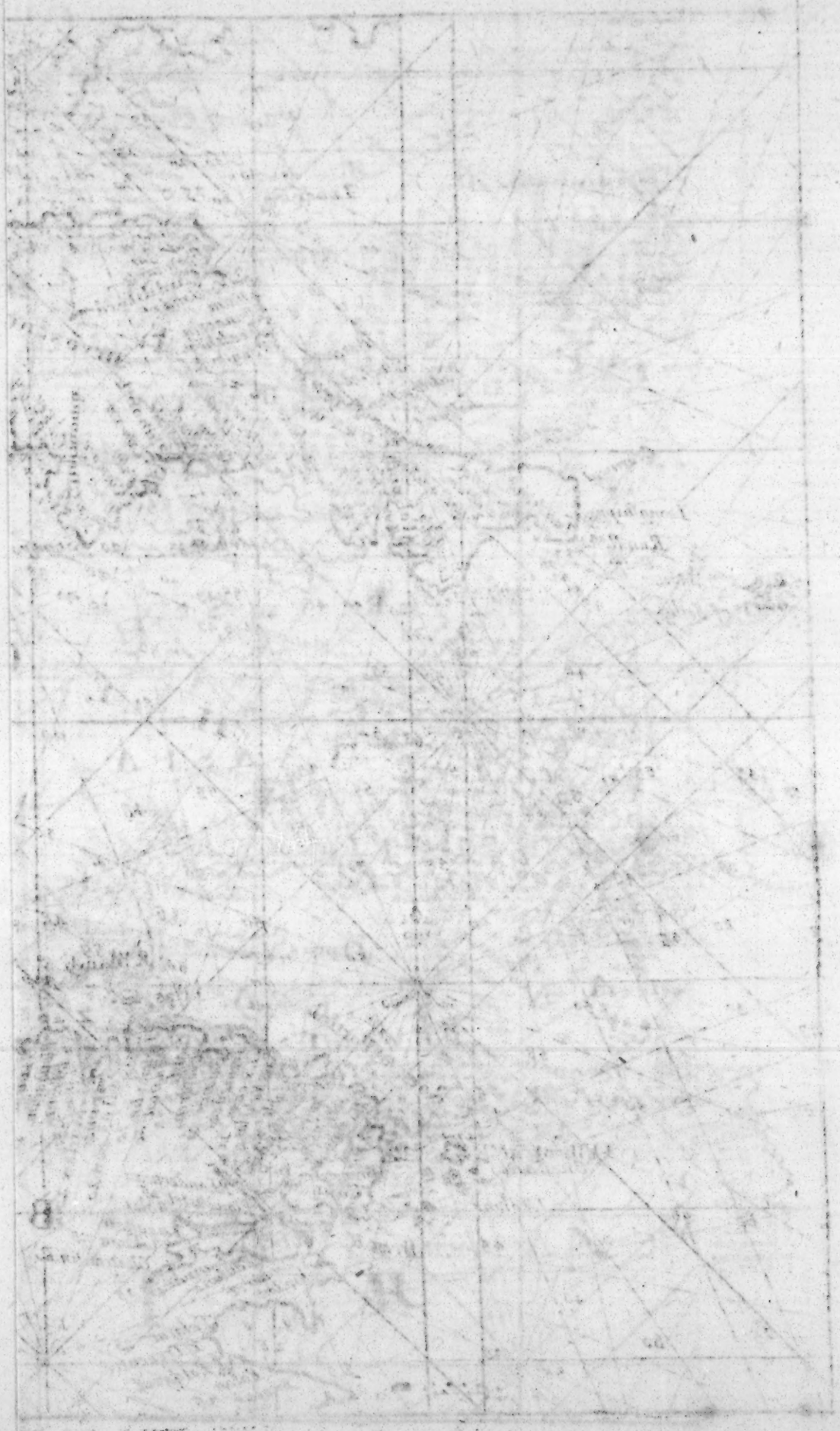
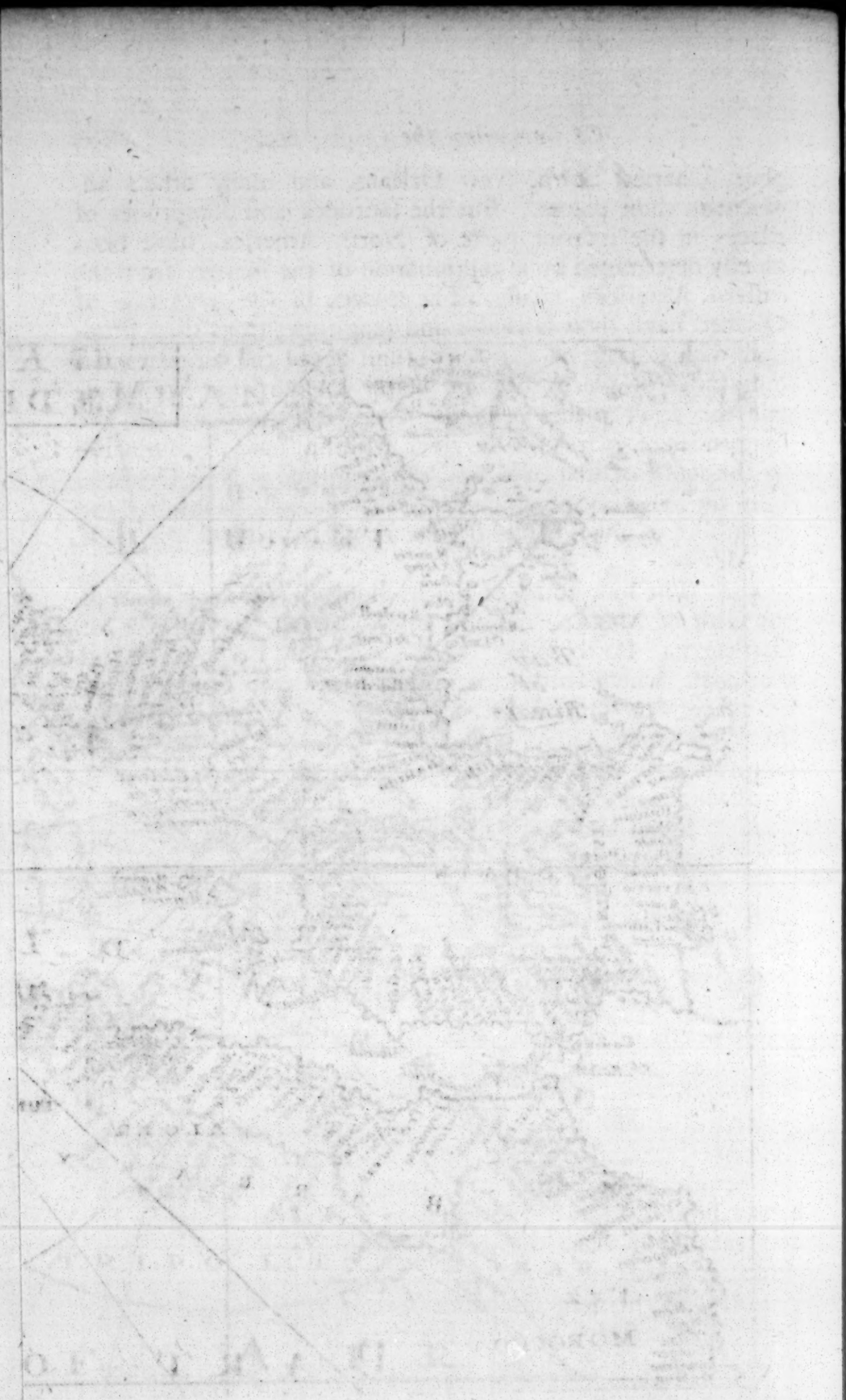
Mud
Clay Ground
Clay Ground
Mud
Mud
Mud

ENGLAND
HAMPSHIRE
DORSETSHIRE
DEVONSHIRE
CORNWALL
KENT
HE
NE
CH
AN
NEL
BRETAGNE
NORMANDIE
PICARDIE
London
Bristoll
Portland
Guernsey
Iersey
Sorlings
Wight I.
Dungeness
N. Foreland
Dover
Deale
Ramsgate
Margate
Sandwich
Tilbury hope
Gravesend
Chatham
Milton
Feversham
Thames Ile
Recolvers
Whitstable
Hampton
Kingsdown
Deale-Dover
St Foreland
Strait of Dover
Ripraps
Beachy head
Eastport
Winchelsea
Dungeness
Rye
Hastings
Fairlee
Dimchurch
Graveling
Fort Mardike
Calais
Fort Nicholas
Fort Rysbank
Fort Ordre
Boulogne
Outreau
Estaples
Montreuil
Berque
Abbeville
Somme R.
Vallery
Bourg d'Ault
Freport
Criecque
Vieux
Calais
Gravelines
Dunkirk
Nieuport
Ostend
Sluys
Middleburg
Flushing
Goree
Helvoetsluys
Rotterdam
Dort
Briel
Maes
Scheveling
Delft
Leyden
Amsterdam
Enchuysen
Medenblick
Horn
Texel
Vlie
Ameland
Schelling
Terschelling
Vlieland
Ameland
Groningen
Embden
Dollart
Lauwers Zee
Zuyder Zee
Zeeland
Walcheren
Cadsand
Sluys
Ostend
Nieuport
Dunkirk
Gravelines
Calais
Boulogne
Ambleteuse
Fort Ordre
Fort Nicholas
Fort Rysbank
Fort Mardike
Graveling



A Chart of the MEDITERRANEAN SEA.

ENG-LAND
London
Thames
IRELAND
Severn
Lizard
Portsm.
Dover
Antwerp
Dunkirk
Calais
Diep
Hauer de Grace
Caen
Roan
St Malo
Nantes
Rochel
Bordeaux
Bayonne
Baiona
Corunna
Viana
Conde
PortaPort
Lisbon
Ubes
Cadiz
Gibraltar
Tangier
Ceuta
Penon Veles
Alboran
Fez
Morocco
Algier
Tunis
Tripoli
Barbary
Part of Africa
Egypt
Spain
Portugal
France
Hungary
Europe
Crim Tartary
Black Sea
Asoff
Constantinople
Greece
Asia
Levant
Sea
Venice
Zara
Ragusa
Candia
Cyprus
Rhodes
Scicily
Malta
Sardinia
Corsica
Majorca
Minorca
Ivica
Formentera
Barcelona
Genoa
Naples
Leghorn
Pisa
Ravenna
Ancona
Messina
Smyrna
Tripoli
Barca
Alexandria
Cairo
Nile R.
Jaffa
Acari
Carmel
Algulo
Scanderon
Tortosa
Tripolis
Sur Tyre
Sidon
Gallipoli
Negropont
Scio
Smirna
Rhodes
Cyprus
Candia
Alexandria
Egypt



phia, Charles Town, New Orleans, and many others adjacent to thoſe places. But the latitudes and longitudes of places in the interior parts of North America, have been chiefly determined by a continuation of the ſurveys from the eaſtern American coaſt. The places in the province of Quebec have their latitudes and longitudes from their ſituation with reſpect to that city; and hence the latitudes and longitudes of places become better known near Montreal, and the other places near the river St. Laurence; and the longitudes of places on the river Miſſiſippi are determinable by the courſe of that river, and the longitude of New Orleans; from the longitude of which place may likewiſe be inferred the longitudes of other places on the northern coaſt of the Gulf of Mexico.

446. The longitudes of places along the ſouthern coaſt of the Gulf of Mexico, may be inferred from the ſituation of Carthagena, Porto Bello, Vera Cruz, and other places on that coaſt, whoſe latitudes and longitudes are known. And the places near to Jamaica, the Havanna, and other parts of the Weſt Indies, may be inferred from the latitudes and longitudes of ſuch adjacent places as have been determined by aſtronomical obſervations.

447. Theſe and many other places are proper ones for making obſervations of the variation of the magnetic needle, from time to time, by a continued ſeries of experiments, until it be diſcovered whether there be any real return of the ſame quantity of variation for the ſame places, amongſt themſelves, or not: and if, by ſuch obſervations, it ſhould appear that the quantity of the variation, after a certain period of time, returns; and the lines of equal variation become as they have been before, for any known year; it will then be very eaſy to predict the variation, and with the greateſt certainty to have perpetual and accurate variation charts of the lands for the purpoſes of geography, and of the ocean for the purpoſes of navigation. But, if no ſuch return ſhould happen, the variation will notwithſtanding be of great uſe in thoſe ſciences, if it be applied as has been before directed.

448. I might here introduce ſeveral other things, which have a near affinity to what has been delivered in the preceding pages, ſuch as the manner of delineating fortified and unfortified towns, and other ſuch places; the manner

of delineating harbours, rocks, ſands, and other places contiguous to the ſea; of rivers, mountainous parts, valleys, and the manner of delineating deſert, incloſed, and woody countries, all which do more particularly belong to that part of geography called topography. But ſhall haſten to the next ſubject which is to be conſidered in this treatiſe; the method of finding the longitude by taking the moon's diſtance from certain of the fixed ſtars, which are called the zodiacal Stars.

449. The daily ſeparation of the moon from amongſt the fixed ſtars, whereby ſhe appears to be removed from ſuch ſtars eaſtward, at the end of every twenty-four hours; and which aſtronomers call the moon's preceſſion according to the order of the ſigns of the zodiac, but her receſſion in reſpect to her coming to the meridian later than the ſtars; was known to the oldeſt obſervers of the motions of the celeſtial bodies. In my New Atlas of the Mundane Syſtem, I have, from the authority of the ancients, ſhewn that this was known to Atreus the king of Argos, ſo early as the time of the Argonautic expedition.

450. Modern aſtronomers apply this apparent diurnal preceſſion of the moon, with peculiar advantages, in the finding of the longitudes of places on land, when no other method can be applied; but on account of the frequent opportunities that happen by this method, it is likewiſe very commodious for application at ſea.

451. According to this method, the moon's apparent diurnal receſſion from the zodiacal ſtars, is the foundation on which the diſcovery of the longitude depends; and as this amounts either to 13 degrees, or ſomewhat more or leſs, on account of the moon's unequal apparent motion through the heavens, this becomes nearly proportional to the whole circumference of the equinoctial line. And becauſe 13 degrees of receſſion anſwer ſomewhat near to the whole circumference 360 degrees; therefore, by proportion, two minutes of a degree receſſion will anſwer nearly to a degree of longitude, or of the equinoctial.

452. The moon's apparent motion through the heavens being a little different at the end of every day, aſtronomers compute the moon's place for either every 6 or 3 hours, throughout the year, and the intermediate motion from the beginning of one three hours to the beginning

ning of the next three hours is the leſs affected by theſe irregularities.

453. The following is an extract from " Ephemerides " des Mouvemens Celeſtes, pour dix années, depuis 1755 " juſq'en 1765, et pour le meridien de la ville de Paris. " Par M. de la Caille.

" Extrait de l'almanac Nautique, pour la fin du mois de " Mai 1754.

" 1754 le 25 Mai.

Regulus.

	Soir	°	′	″	
"	4	39	48	50	Diſtance au bord eclaire de la Lune.
"	5	39	14	51	
"	6	38	40	54	
"	7	38	6	58	
"	8	37	33	3	
"	9	36	59	9	

" Parallax log. 2 5390

" Then follow other diſ-
" tances, for May 26.

Le 27 Mai.

Le Soleil.

Matin	°	′	″	
6	62	57	2	Diſtance au bord eclaire de la Lune.
7	63	29	20	
8	64	1	40	
9	64	34	1	
10	65	6	22	
11	65	38	44	
12	66	11	6	

Parallax log. 3 5426

Then follow other diſtances, for other ſtars."

This was the firſt plan of l'Almanac Nautique that I have heard of.

454. In his preceding pages, he has ſhewn very eaſy methods of finding the effects of refraction and parallax, which are the principal difficulties to be overcome in calculating the obſervations when they are made; but after all, he was not ſatisfied with this method. This appears from the difficulties he complains of, page 31 of the ſame work.

455. This complaint of the Abbé's, was founded on his experience. He found his obſervations would not agree. He ſettled his time by calculating his hour angle from a ſingle obſervation. No aſtronomer at that time ſuſpected any allowance ſhould be made for the ſpheroïdal horizon. And, ſpeaking generally, neither the determination of the hour angle by the Abbé, nor by any other aſtronomer from a ſingle altitude taken above the horizon of the ſea, the latitude of the place, and declination; could poſſibly be true under all caſes or directions, without due allowance for the deviation of gravity: the effects of which, I have before ſhewn in this work. And therefore it is no wonder that the

obſervations for the longitude at ſea, by the moon's diſtance from the ſun and ſtars, may not have anſwered, when it has been tried and practiſed by the very beſt aſtronomers.

456. In order to the diſcovery of the longitude by the ſun and moon, according to this plan or method, three obſervations are requiſite; and they are to be made at or near the ſame inſtant of time, namely, the altitude of the ſun, the altitude of the moon, and the diſtance of the ſun and moon's neareſt limb. But, in order to the diſcovery of the longitude by a ſtar and the moon, although three obſervations would be enough, could they be depended on, namely, the altitude of the ſtar, the altitude of the moon, and the diſtance of the moon and ſtar; it is uſual to admit an obſervation of the ſun either before or after the obſervation of the ſtar and moon, in order to ſet or adjuſt a watch to either mean or ſolar time, and know thereby the more readily at what mean or ſolar time the diſtance of the moon and ſtar is taken in the night time; which, for want of an horizon in the night time, and on account of the ſtar's right aſcenſion being different from the ſun's, could not ſo readily, nor perhaps ſo accurately, be known in all caſes; eſpecially when an excellent watch that may be depended on is uſed. But this introduction of the uſe of a watch is not through abſolute neceſſity, eſpecially when there is a tolerably good appearance of the horizon in the night; and the reſt will be the calculation of a few additional triangles.

457. The three obſervations taken, or ſuppoſedly taken, near the ſame inſtant of time, are called the three cotemporary obſervations, of which the diſtance of the ſun and moon, or of moon and ſtar, are the principal; becauſe, as has been before noted, if there happens to be an error of but two minutes of a degree in one of theſe, the obſerver may err in his longitude a degree, however correct his other obſervations and calculations may be; and this, at the equinoctial, will produce an error of ſixty geographical miles; but leſs towards the poles, in the proportion of the radius to the coſine of the latitude of the place of obſervation.

458. If the obſervation is made in the night time, the ſemidiameter of the moon only muſt be known; but if the obſervation be in the day time, the ſemidiameter of the ſun is likewiſe to be known: and although it be not ſtrictly

true

true in theory, it will in moſt practical caſes be inſenſibly different therefrom, to add the apparent ſum of the ſun and moon's ſemidiameters to the obſerved diſtance of their neareſt limbs, and to take the ſum for the obſerved diſtance of the ſun and moon's centres.

459. By the apparent ſum of the ſun and moon's ſemidiameters is meant, what the ſemidiameter of the ſun and of moon do meaſure, at their elevation above the horizon, when the obſervation is made. The ſun's ſemidiameter can be but very little augmented by ſuch elevation; but the moon's ſemidiameter is augmented thereby; for which a table may be made, ſhewing, how many ſeconds the moon's ſemidiameter is leſſened by her altitude.

460. The nearneſs of the moon to the earth, makes her appear to a ſpectator on the earth's ſurface lower than ſhe is, except when ſhe is in the zenith; this greateſt depreſſion is when ſhe is in the horizon, and therefore called her horizontal parallax. As ſhe advanceth in altitude, that parallax diminiſhes, and nearly in the proportion of the radius to the coſine of the altitude. Such parallax at any altitude is called her parallax in altitude. Both of theſe forementioned effects may be found, either by calculation, or by the parallactic triangle, or by tables for thoſe purpoſes.

461. As the moon's parallax in altitude makes her appear leſs elevated than ſhe really is, ſo the refraction makes her appear more elevated than ſhe really is; and therefore the refraction being ſubtracted from the moon's obſerved altitude, gives her true altitude, except the effect produced by her parallax in altitude.

462. The ſun being at a much greater diſtance from the earth, has thereby a much leſs horizontal parallax. Some make the ſun's horizontal parallax $8\frac{2}{3}''$, others differ therefrom. But whatever it be; as obſerved angular diſtances of the ſun and moon cannot be depended on, when the ſun is near the horizon, on account of the horizontal vapours, and the change of the ſtate of theſe vapours is not eaſily to be known; and farther, becauſe no inſtrument can take the angular diſtance of the ſun and moon, without erring a few ſeconds of a degree; therefore it can anſwer no practical purpoſe of introducing corrections for the ſun's parallax in altitude; ſeeing it is agreed on to amount to but 4 or 5 ſeconds

ſeconds of a degree, at a third part of the greateſt elevation in any latitude; and at other elevations leſs, which makes it ſtill the more inconſiderable, and needleſs to be uſed in this method of finding the longitude.

463. The dip of horizon is occaſioned by the obſerver's being above the ſurface of the ſea: was his eye in that ſurface, it would not ſee the ſun or any other celeſtial body at riſing, ſo ſoon as if it was ten, twenty, thirty, or more feet, above that ſurface; and the parallactic angle, by which the eye at ſuch an elevation can ſee under the horizon that would appear to an eye at the ſurface of the water, is called the dip of horizon, and uſually allowed for by a table for that purpoſe.

464. When the reaſons and demonſtrations of mathematical theorems cannot be underſtood by perſons who want the uſe of them for their own affairs of buſineſs and life; or when, through an indolent diſpoſition, they do not chooſe to take the trouble of going through ſuch demonſtrations, they frequently chooſe to be content with the manner of applying them. This is a practice much ſpoke againſt and diſapproved of by ſome theoriſts, who would have people either wholly deprived of the uſe of ſcience, or thorough proficients as they call it; certainly this is falſe reaſoning. Undoubtedly it would be beſt to have the theory as well as the practice of any ſcience of this kind; but where there is no room for the former, and the latter is indiſpenſably requiſite, it ſhould not be ſet aſide. I could produce the opinion of a great aſtronomer, that aſtronomers might be able, but mariners never would be able, to practiſe the longitude at ſea by the moon, on account of the difficulties in making and computing the obſervations; nevertheleſs, a few years practice has ſhewn the contrary. And I could produce a ſecond opinion, that rewards are due to perſons who can make it plain and eaſy.

465. From ſuch conſiderations as theſe, and with a deſire to aſſiſt perſons but little acquainted with mathematical affairs, when the Engliſh Nautical Ephemeris began in 1767; I made a Formula for taking off that part of the calculation of the longitude problem, which relates to the effects of refraction and parallax, by the uſe of the requiſite tables, in the ſecond volume of the Nautical Ephemeris, 1767.

This

Greatest Altitude . . . } N.° in Table I. 2 . . . . "

Distance . . . Log. Corecant _____ . . . . .

. . . Common Log. 1 _____ . . . . . = A =

Distance & Least Altitude N.° in Table II. = B =

If Distance above 90°. Add A & B. otherwise Subtract gives =

Distance Add _____ . . .

Hor. Par. . . Prop. Log. . . . . D = _____ . . .

⊙'s Alt.d . . Log. Corec.t . . . . .

N.° D. . . Log. Sine. _____ . . . . . ° ' "

Sum a Prop.l Log.m 2 _____ . . . . . First Arc = . . .

Hor. Par. . . Prop.l Log. .

D's Alt.d . . Log Corec.t . . . . .

° ' "

N.° D. . . Log. Tang.t _____ . . . . .

Sum a Prop.l Log.m 2 _____ . . . . . Second Arc = _____ . . .

Sum of the Arcs if D above 90°. otherwise the Difference C = . . .

Sum of C & D. if First Arc is least & D under 90°. D = _____ . . .

otherwise the Difference gives E = . . .

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A Calculation at Large

Given the Ship's Lat.d by Observation 34°.17 N. & Long.d 17°.46 W. by acc.t
Dist.ce ☉ & ☽'s nearest Limbs obs.d 73.44.27. Semid.r 31'.36" } by Watch
Alt.e of ☉'s l.r L.b above the Sea obs.d 22.3. height of the Eye 18 Feet. } 4h.50'.57"
Alt.e of ☽'s l.r L.b above the Sea obs.d 80.41. height of the Eye 18 Feet.
Alt.e of ☉'s l.r L.b above the Sea obs.d 19.13 a Correct Observ.n at 5h.4'.38"
April 4th 1767 afternoon. Dip of hor.n is 4' & ☉'s Semid.r 16'. Req.d the Long.d ?

The Diagram-Plate: Shews the Solution of Prob.m One for the Time, the Other for the Latitude & the Time

2.dly For the Refraction.

22°.15' } = 2.2293
80.53 }
74.16.3" = 10.0165
A = 176" = 2.2458
74°.16' } B = 32"
22.15 }
144" = 2'.24"
See Formula 74°.16'.3"
D = 74°.18'.27"

3.dly For the Time at Greenwich.

3h. = 73°.1'.27"
P = 74.11.58
1°.10'.31" = .4070
3h. = 73°.1'.27"
6h. = 74.28.50
1°.27'.23" = .3138
2h.25.14" = .0932
See Ephem.s 3h
G = 5h.25'.14"

& For the Parallax.

56'.12" = .5055
22°.13. = 10.4223
74.18.27 = 9.9835
1.st Arc = 22'.5" = 0.9113
56'.12" = .5055
80°.53. = 10.0055
74.18.27 = 10.5512
2.d Arc = 15'.36" = 1.0622
C = 6'.29"
D = 74°.18'.27
E = 74°.11'.58"
74°.18' } = 0" }
10" = 0" } = F = 0"
6" = 0" }
P = 74°.11'.58"
See Formula.

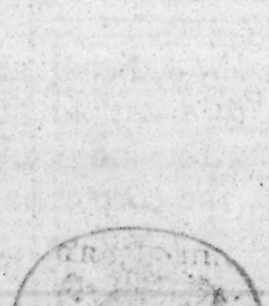
4.thly For the Time at the Ship.

See Diagram.
Lat.d = 34°.17'
PZ = 55.43
☉'s Decl.n = 5°.48'
PS = 84.12
☉'s Alt.d = 19°.22'
ZS = 70.38

PZ = 55°.43 Co. ar. .082882
PS = 84.12 Co. ar. .002229
ZS = 70.38 9.984397
2) 210.33 Sine 9.754595
74°.44 = 105.16 Sine 19.824103
34.38
35°.15' = Cosine = 9.912051
2
70°.30' = 4h.42'.00" by Observation.
5.4.38 by the Watch.
Watch 22'.38" too Fast.
4h.50.57
S = 4h.28'.19"

5.thly For the Longitude

Time of 3 Obs.s at Greenwich 5h.25'.14"
Time of 3 Obs.s at the Ship 4.28.19
L = 14°.14' W = 0h.56'.55"
Ship West of Greenwich.



This Formula is an eaſy application of the tables which had been formed by other perſons. However, it has been approved of; and ſeveral journals of obſervations for the longitude have been kept by it, in important voyages, to the ſatisfaction of ſkilful and experienced mariners. When the longitude plan came forth before the public, and I was publicly recommended to teach it, it appeared to me very improper to be ſingular. I took the firſt theorem that came to hand, and explained it; had it been any other, probably it might have anſwered the ſame purpoſe.

466. This Formula was contrived by me ſoon after the publication of the Nautical Ephemeris; and, in the beginning of the year 1769, I added to it ſhort directions for finding the longitude by help of the nautical almanac and the requiſite tables, thereby making a thin octavo, quite convenient and fit for the pocket, wherein the operations and calculations for the longitude might be eaſily and neatly inſerted during the courſe of one or more voyages, and from thence either entered into the ſhip's journal, or referred to as occaſion might require. And it may be now had of me for ſuch purpoſes.

467. In making the calculation for the longitude by the moon, the greateſt difficulty is in finding the effects of refraction and parallax, as beforementioned; this ſeems to have been a ſtumbling-block to learners; and it is obvious, that ſome of the precepts which have been delivered for overcoming theſe difficulties, have appeared prolix and tedious to mathematicians themſelves; and therefore it is not to be wondered at, that ſuch ſhould appear difficult and tedious to thoſe who have but little inſight into thoſe intricate affairs.

468. In the year 1766, being aſked by a gentleman, who ſince that time has received no little ſhare of deſerved honour and profit from the longitude problem, what was my thought concerning the correction of refraction and parallax, I gave the following anſwer: "Little differences in "the heavens ariſing from the effects of refraction and "parallax, give a fair opportunity for introducing the pro- "perties of plane triangles, and ſolving the problem thereby "much more eaſily than by ſpherical triangles." The anſwer was; "But they are ſpherical triangles, and their ſolu-

"tion

" tion otherwiſe will be erroneous." The above expreſſion of mine is conformable with what I have delivered at the end of the 330th ſection of this work. It is what I have taught in various parts of ſcience with all deſirable ſucceſs. It is what the moſt curious aſtronomers have admitted, in correcting the little inequalities which ariſe amongſt the celeſtial bodies from time to time, by their unequal motions through the heavens; and it is what the beſt mathematicians, in every country and in every age, have admitted in the conſtruction of their niceſt practical theorems. The elaborate theorems which have ariſen from other principles, are to be ſeen in the works of thoſe who have invented and publiſhed them. It is ever ſince the abovementioned time, that another method for finding the effects of refraction and parallax, and ſuch an one as is ſufficiently exact for practical caſes, has been known to me, deriveable from thoſe plain and ſelf-evident principles; but before this time, I have had no proper opportunity of publiſhing it in print.

469. If Z repreſents the zenith, S the ſun, and M the moon; S Z is the ſun's coaltitude, M Z the moon's coaltitude, and S M the obſerved diſtance of ſun and moon; having theſe three ſides of the oblique angled ſpherical triangle given, I applied a method, which was invented by me fifteen years ſince, and thereby found the angles M and S by inſpection, and near enough to the truth for avoiding any conſiderable error that might render it uſeleſs in taking off the effects of parallax and refraction. This was not by means of a planiſphere, although it may be readily done that way and many others; but by help of a new globular ſector, fitted up properly and made very portable, and ready for practice.

470. And ſeeing that the moon's parallax in altitude depreſſeth the moon, but the refraction elevates her, and the parallax in altitude is always greater than the refraction; the joint effects of parallax and refraction do elevate the moon in the vertical circle as from M to *m*, by the difference between the parallax in altitude and the refraction. But the ſun's horizontal parallax not exceeding 9″, and his parallax in altitude being leſs than 9″, and frequently inconſiderable, it may be rejected; and the whole ſenſible effect of refraction and parallax in the vertical circle is produced by the refraction, which, being taken off, brings the ſun to *s*; and joining the

the points *m* and *s*, by an arch of a great circle, *m s* is the true diſtance of the ſun and moon cleared from refraction and parallax, at leaſt, correct enough for practical purpoſes.

471. From theſe plain principles, an eaſy method ariſeth of ſolving the problem, when the angles M and S are ſo found, by inſpection, by a compariſon only of the ſun's refraction with the obſerved diſtance; and by a compariſon of the difference between the moon's refraction and her parallax in altitude, with the ſame obſerved diſtance. For, if a perpendicular be dropt from *m* to *n*, and the arch of a great circle be drawn from *s* to *n*, then will *s n* be very nearly equal to *s m*, on account of the length S M and the ſhortneſs of M *m*. And by the ſame way of reaſoning, if a perpendicular be dropt from S to *p*, the length *m p* will be very nearly equal to *m* S. And finally, the whole correction depends upon the little triangles M *m n*, and S *s p*, both of which may be conſidered and treated as plane triangles; the hypotheneuſe of the one being M *m*; and of the other S *s*; and the two little differences M *n* and *s p* being thoſe quantities which make the true diſtance of the ſun and moon differ from the obſerved diſtance. And the like method is applicable for the correction of the diſtance between the moon and a ſtar.

472. And hence it follows that a ſmall diagram being made with the hand, ſomewhat conformable with the two coaltitudes, and alſo with the angles S and M, will ſhew whether ſuch little differences are either additive or ſubtractive, and thereby give the practitioner ſuch an inſight into the reaſons of the operation, as cannot be dictated by any precepts only, or tables. And hence it appears that the problem has but two principal parts, the finding of the angles of a ſpherical triangle having its three ſides given, which I have found a method of performing by inſpection; and ſecondly, the finding of the baſe of a plane triangle, when the hypotheneuſe and the angle next the baſe are given, which may be eaſily done by Gunter's lines of numbers and ſines, or for want of them by the parallactic triangle, or by the logarithms.

473. Although a method be here mentioned of finding the angles M and S by inſpection, and that be ſufficiently exact for practical caſes, if any chooſe to compute thoſe an-

gles by the logarithms, it will be but an addition to the computation; and those angles may be found by several of the theorems inserted before under the name of spherical triangles, in this work.

474. The method of finding the moon's parallax in altitude having her altitude and horizontal parallax has been already shewn; but if that be thought tedious, it may be affected by a table of the parallax in altitude to seconds, and so to each minute of the horizontal parallax; but to take off the introduction of so large a table as would be wanted for that method, I have given the parallax in altitude for the horizontal parallax 53′ only, and by applying the parallactic triangle, the additional parallax in altitude to any other horizontal parallax, may be found as follows:

EXAMPLE.

What is the parallax in altitude, for the altitude 28° 0′, and the horizontal parallax 58° 0′?

CALCULATION.

				′	″
Parallax to horizontal parallax	—	53′	is	46	47
Additional parallax for overplus	—	5′	is	4	26
Horizontal parallax required	—		is	51	13

ANOTHER.

What is the parallax in altitude, for the altitude 75° 28′, and the horizontal parallax 55′ 28″?

CALCULATION.

				′	″
Parallax to horizontal parallax	53′		is	13	15
Additional parallax for overplus	2′	28″	is		38
Horizontal parallax required	—			13	53

In this method of taking out the parallax in altitude, answering to any altitude and horizontal parallax given; the parallax in altitude agreeing with the horizontal parallax 53′,

is

Alt.d	Parallax		Alt.d	Parallax		Alt.d	Parallax	
°	′	″	°	′	″	°	′	″
0	53.	0	30	45.	54	60	26.	30
1	52.	50	31	45.	26	61	25.	42
2	52.	57	32	44.	57	62	24.	53
3	52.	55	33	44.	27	63	24.	4
4	52.	52	34	43.	57	64	23.	14
5	52.	48	35	43.	25	65	22.	24
6	52.	43	36	42.	52	66	21.	33
7	52.	36	37	42.	19	67	20.	42
8	52.	29	38	41.	46	68	19.	51
9	52.	21	30	41.	11	60	19.	0
10	52.	12	40	40.	36	70	18.	8
11	52.	2	41	40.	0	71	17.	16
12	51.	51	42	30.	23	72	16.	23
13	51.	30	43	38.	46	73	15.	30
14	51.	26	44	38.	8	74	14.	37
15	51.	12	45	37.	29	75	13.	43
16	50.	57	46	36.	49	76	12.	40
17	50.	41	47	36.	9	77	11.	55
18	50.	24	48	35.	28	78	11.	1
19	50.	7	49	34.	46	79	10.	7
20	49.	48	50	34.	4	80	9.	12
21	49.	29	51	33.	21	81	8.	18
22	49.	9	52	32.	38	82	7.	23
23	48.	47	53	31.	54	83	6.	29
24	48.	25	54	31.	9	84	5.	34
25	48.	2	55	30.	24	85	4.	39
26	47.	38	56	29.	33	86	3.	43
27	47.	13	57	28.	52	87	2.	48
28	46.	47	58	28.	5	88	1.	53
29	46.	21	59	27.	18	89	0.	59

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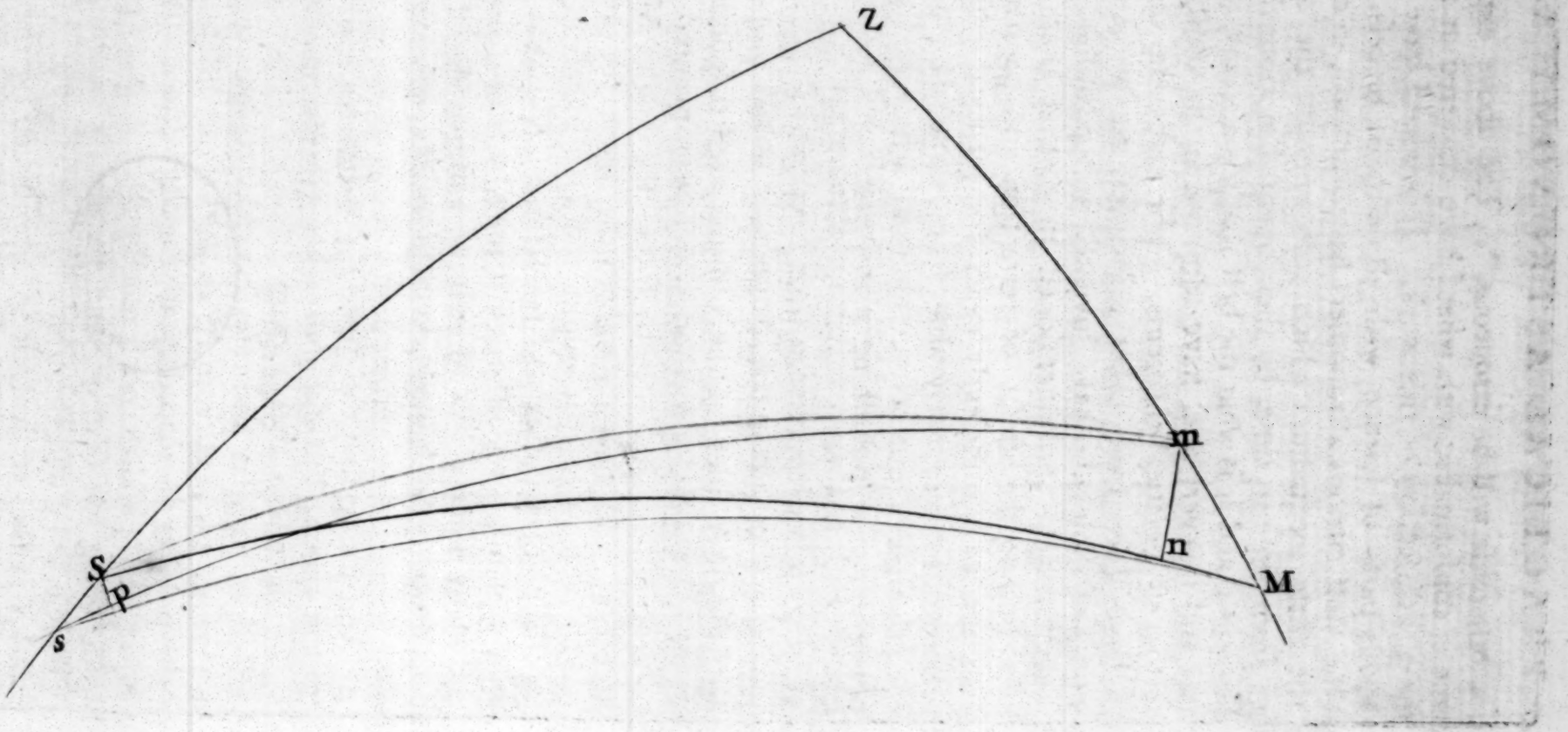
Z
m
n
S
p
s
M

B

is firſt taken out of the table; and having taken the difference between 53′ and the given horizontal parallax, and turned that difference into ſeconds, the number of ſeconds is found in the line A B of the parallactic triangle, and guiding your eye in the direction of the concentric arches, until you come oppoſite to the altitude in the limb, perpendicularly beneath in the line C B is the additional parallax, which turned into minutes, and added to the former, is the parallax required. As a larger parallactic triangle would ſhew this plainer; it is delineated, with improvements.

475. Theſe things being underſtood, the method of finding the effects of refraction and parallax in the diſtance of the ſun and moon, or moon and ſtar, will be eaſy, as may be illuſtrated; thus,

EXAMPLE.

	°	′	″
Given the ſtar's coaltitude obſerved	65	12	0
the moon's coaltitude obſerved	77	30	9
Diſtance of their centres obſerved	51	28	35
Moon's horizontal parallax		56	15

Required the correct diſtance of the ſtar and moon's centres, cleared from refraction and parallax?

CALCULATION.

			°	′	″
Angle at the ſtar by calculation or inſpection			93	39	0
Angle at the moon			68	8	0
Moon's parallax in altitude	3287″	or		54	47
Summer refraction for the moon	222	or		3	42
Difference	3065	or		51	5
Moon's parallax in altitude	3287″	or		54	47
Winter refraction for the moon	249″	or		4	9
Difference	3038″	or		50	38
Star's refraction ſummer	106″	or		1	46
winter	120″	or		2	0
Difference	14″				

From theſe compariſons, it is evident, that the varying ſtate of the refractions, eſpecially near the horizon, cannot but affect the correction for the diſtance of the centres; and as this may not eaſily be allowed for, without an error of

a few ſeconds, pretenſions to abſolute accuracy cannot but be uſeleſs.

In the preſent example, taking things as they are; it will be,

		°	′		″		″
As radius	: coſine	93	39	: :	106	:	7
or As radius	: coſine	93	39	: :	120	:	8
		The difference is					1

And again,

		°	′		″		″
As radius	: coſine	68	8	: :	3065	:	1142
or As radius	: coſine	68	8	: :	3038	:	1132
		The difference					10

This is a farther proof that, the refractions being variable, the corrected diſtances of the centres cannot be ſo certain as they otherwiſe would be; and therefore it is requiſite in ſome caſes, in order to have the diſtance of the centres correct, to have correct tables of refraction, as well as of the parallax in altitude; eſpecially, when the arch of the great circle joining the centres of the ſun and moon, or ſtar and moon, paſſeth near the zenith of the place of obſervation.

476. But, taking things as they occur in the preſent example; from 1142″ take 7″, remains 1135 or 18′ 55″, the effect of refraction and parallax according to the ſummer refraction. And, from 1132″ take 8″, remains 1124″ or 18′ 44″, the effect of refraction and parallax according to the winter refraction. So the correct diſtance of the ſtar and moon's centres, according to the ſummer refraction, is 51° 9′ 40″, and according to the winter refraction 51° 9′ 51″, which latter number is the exact number found by another kind of proceſs at page 22, of the requiſite tables to the nautical almanac. But it may be obſerved, that as an error of 2 minutes of a degree in the diſtance, may produce an error of 60 geographical miles in longitude at the equinoctial; ſo an error, or difference, ſuch as the above, of 11″ of a degree, in the angular diſtance of the luminaries, ariſing from the difference of refraction, may there produce an error of $5\frac{1}{2}$ geographical miles; and greater errors in the corrected angular diſtances of the luminaries may happen, where the effects of refraction cannot be better aſcertained. Hence it appears, not only how tables for finding the effects of refraction and parallax may very eaſily be made; but how the effects may be found from original principles,

very

very readily, without tables. And this improvement could gain no admittance.

477. A TABLE, ſhewing how many minutes and ſeconds of time, the whole diameter of the ſun is apparently paſſing over a line coincident with the plane of the meridian of a place, throughout the year.

Day	January	February	March	April
	′ ″	′ ″	′ ″	′ ″
1	2 22	2 16	2 11	2 9
9	2 21	2 14	2 10	2 10
17	2 19	2 12	2 9	2 10
25	2 17	2 11	2 9	2 11

Day	May	June	July	Auguſt
	′ ″	′ ″	′ ″	′ ″
1	2 12	2 17	2 18	2 13
9	2 13	2 18	2 17	2 12
17	2 15	2 18	2 16	2 10
25	2 16	2 18	2 15	2 9

Day	September	October	November	December
	′ ″	′ ″	′ ″	′ ″
1	2 9	2 9	2 15	2 21
9	2 8	2 10	2 16	2 22
17	2 8	2 11	2 18	2 23
25	2 9	2 13	2 20	2 22

This table will be very uſeful in taking the ſun's meridional tranſit. For, if the time of the contact of the limb be obſerved, and half the above time of duration be allowed, the time of central tranſit is known.

478. It is uſual in moſt books of aſtronomy to give tables of the declination of the ſun to every day in the year, for the Biſſextile year, and alſo for one, two, and three years after it; but as this makes the tables of the ſun's declination large, I ſhall here ſhew a very ſhort and eaſy method, whereby the declination of the ſun may be had within a minute of a degree, or within half a minute of a degree in many caſes, by tables of that declination to every fourth day; the principle on which the method is founded, is as follows. If 1 be added to the number 5, the ſum is 6, and its half is 3; and if 1 be added to 3, the ſum is 4, and its half is 2. Again, if the number 5 be added to 9, the ſum

is

is 14; and the half of 14 is 7; and 7 added to 5 makes 12, whose half is 6. Again, 7 and 9 make 16, whose half is 8. This gives the most easy method of contracting all tables whatsoever, that may be contracted without sensible error. This may be exemplified, by supposing the declination of the sun given for any two days, which are four intermediate days distant from each other, as in the following examples.

Example.

Required the sun's declination May 26th and 28th, in a Bissextile year?

		°	′	″	
For May 25,	declination	21	6		per table.
29,		21	45		
	Sum	42	51		
For 27,	declination	21	25	30	
	true declination	21	26	15	
	Error			45	

Again,

		°	′	″	
For May 25,	declination	21	6		per table.
27,	by the medium	21	25	30	
	Sum	42	31	30	
For May 26,	declination	21	15	45	
	true declination	21	16	20	
	Error			35	

Again,

		°	′	″	
For May 27,	declination	21	25	30	
29,		21	45	0	
	Sum	43	10	30	
For May 28,	declination	21	35	15	
	true declination	21	35	50	
	Error			35	

Another.

Required the declination for 26th and 28th of March?

		°	′	″	
For March 25,	declination	2	9		per table.
29,		3	42		
	Sum	5	51		
For March 27,	declination	2	55	30	
	true declination	2	55	45	
	Error			15	

Again,

Again,		°	′		
For March 25,	declination	2	9		per table.
27,	by the medium	2	55	30	
	Sum	5	4	30	
For March 26,	declination	2	32	15	
	true declination	2	32	19	
	Error			4	

Again,		°	′		
For March 29,	declination	3	42		per table.
27,	by the medium	2	55	30	
	Sum	6	37	30	
For March 28,	declination	3	18	45	
	true declination	3	19	9	
	Error			24	

ANOTHER.

Required the declination for 26th and 28th of June?

		°	′		
For June 25,	declination	23	24		per table.
29,		23	13		
	Sum	46	37		
For June 27,	declination	23	18	30	
	true declination	23	19	13	
	Error			43	

Again,		°	′		
For June 25,	declination	23	24		per table.
27,	by the medium	23	18	30	
	Sum	46	42	30	
For June 26,	declination	23	21	15	
	true declination	23	21	40	
	Error			25	

Again,		°	′		
For June 29,	declination	23	13		per table.
27,	by the medium	23	18	30	
	Sum	46	31	30	
For June 28,	declination	23	15	45	
	true declination	23	16	15	
	Error			30	

From theſe examples, it appears, that by ſmall tables of the ſun's declination for a Biſſextile year, and the three following

following years, the declination of the sun may be had by halfing only, for any intermediate day, frequently without an error of half a minute of a degree.

479. A TABLE of the declination of the sun, for a Bissextile year. This table will answer for 1776, 1780, 1784, &c.

Day	January	February	March	April
	° ′	° ′	° ′	° ′
1	23 2	17 8	7 15	4 52
5	22 39	15 57	5 43	6 23
9	22 8	14 42	4 9	7 53
13	21 31	13 24	2 35	9 21
17	20 46	12 1	1 0	10 46
21	19 56	10 36	0 34 N.	12 8
25	19 0	9 8	2 9	13 27
29	17 58	7 38	3 42	14 43

Day	May	June	July	August
	° ′	° ′	° ′	° ′
1	15 19	22 10	23 5	17 52
5	16 29	29 39	22 45	16 48
9	17 34	23 1	22 18	15 40
13	18 35	23 16	21 45	14 28
17	19 31	23 25	21 6	13 12
21	20 21	23 28	20 22	11 53
25	21 6	23 24	19 32	10 31
29	21 45	23 13	18 36	9 6

Day	September	October	November	December
	° ′	° ′	° ′	° ′
1	8 1	3 30	14 43	21 58
5	6 33	5 3	16 57	22 30
9	5 2	6 35	17 7	22 56
13	3 31	8 5	18 13	23 14
17	1 58	9 34	19 13	23 25
21	0 24	11 0	20 8	23 28
25	1 9 S.	12 24	20 57	23 24
29	2 43	13 45	21 39	23 12

481. A TA-

480. A TABLE of the declination of the ſun, for the firſt year after a Biſſextile year. This table will anſwer for 1777, 1781, 1785, &c.

Day	January	February	March	April
	° ′	° ′	° ′	° ′
1	22 58	16 55	7 20	4 46
5	22 33	15 44	5 48	6 18
9	22 2	14 28	4 15	7 48
13	21 23	13 8	2 41	9 16
17	20 37	11 45	1 6	10 41
21	19 46	10 19	0 29 N.	12 3
25	18 48	8 51	2 3	13 23
29	17 45		3 37	14 39

Day	May	June	July	Auguſt
	° ′	° ′	° ′	° ′
1	15 15	22 8	22 6	17 55
5	16 25	22 37	22 46	16 52
9	17 31	23 0	22 20	15 44
13	18 32	23 16	21 47	14 32
17	19 28	23 25	21 9	13 17
21	20 18	23 28	20 24	11 58
25	21 4	23 24	19 35	10 36
29	21 43	23 14	18 40	9 11

Day	September	October	November	December
	° ′	° ′	° ′	° ′
1	8 7	3 24	14 39	21 56
5	6 38	4 57	15 53	22 29
9	5 8	6 29	17 3	22 55
13	3 36	8 0	18 9	23 13
17	2 3	9 29	19 10	23 24
21	0 30	10 55	20 5	23 28
25	1 4 S.	12 19	20 54	23 24
29	2 38	13 40	21 37	23 13

481. A TABLE of the declination of the ſun, for the ſecond year after a Biſſextile year. This table will anſwer for 1778, 1782, 1786, &c.

	January	February	March	April
Day	° ′	° ′	° ′	° ′
1	22 59	16 59	7 26	4 41
5	22 35	15 48	5 54	6 12
9	22 6	14 33	4 21	7 42
13	21 15	13 13	2 47	9 10
17	20 40	11 51	1 12	10 36
21	19 49	10 25	0 23 N.	11 58
25	18 52	8 57	1 57	13 18
29	17 50	———	3 31	14 34

	May	June	July	Auguſt
Day	° ′	° ′	° ′	° ′
1	15 11	22 7	23 7	17 59
5	16 21	22 36	22 48	19 56
9	17 27	22 59	22 22	15 49
13	18 28	23 15	21 49	14 37
17	19 24	23 25	21 11	13 22
21	20 15	23 28	20 27	12 3
25	21 1	23 25	19 38	10 41
29	21 41	23 15	18 43	9 17

	September	October	November	December
Day	° ′	° ′	° ′	° ′
1	8 12	3 19	14 34	22 54
5	6 44	4 52	15 49	22 27
9	5 13	6 24	16 59	22 53
13	3 42	7 54	18 5	23 12
17	2 9	9 23	19 6	23 24
21	0 36	10 50	20 2	23 28
25	0 58 S.	12 14	20 51	23 25
29	2 32	13 35	21 34	23 14

482. A TA-

482. A TABLE of the declination of the ſun, for the third year after a Biſſextile year. This table will anſwer for 1775, 1779, 1783, 1787, &c.

Day	January	February	March	April
	° ′	° ′	° ′	° ′
1	23 0	17 3	7 32	4 35
5	22 37	15 53	6 0	6 7
9	22 6	14 37	4 27	7 37
13	21 28	13 11	2 52	9 5
17	20 43	11 56	1 18	10 30
21	19 52	10 30	0 17 N.	11 53
25	18 56	9 2	1 52	13 13
29	17 53		3 25	14 29

Day	May	June	July	Auguſt
	° ′	° ′	° ′	° ′
1	15 6	22 4	23 8	18 3
5	16 17	22 34	22 49	17 0
9	17 23	22 57	22 23	15 53
13	18 24	23 14	21 52	14 41
17	19 21	23 24	21 14	13 26
21	20 12	23 28	20 30	12 8
25	20 58	23 25	19 41	10 46
29	21 38	23 15	18 47	9 22

Day	September	October	November	December
	° ′	° ′	° ′	° ′
1	8 17	3 13	14 29	21 51
5	6 49	4 46	15 44	22 25
9	5 19	6 18	16 55	22 52
13	3 47	7 49	18 1	23 11
17	2 15	9 18	19 2	23 23
21	0 42	10 45	19 58	23 28
25	0 52 S.	12 9	20 48	23 25
29	2 26	13 30	21 32	23 14

483. I have delivered this method as a ſpecimen of a general one, which may be extended to any other tables whatſoever. It is generally objected that tables frequently are copious and voluminous: but by this method they may be made very conciſe, and much leſs ſubject to the errors which frequently introduce themſelves in the

beſt tabular publications. And this may be frequently done by this method, without the application of the method of ſecond differences.

484. In a table of the ſun's place in the ecliptic, the numbers keep nearer to an equality throughout the year; and therefore the biſection will give the intermediate days the more exactly. Here follows a table of the ſun's place in the ecliptic for a Biſſextile year, nearly.

Day	January S. ° ′	February S. ° ′	March S. ° ′	April S. ° ′
1	9 10 43	10 12 16	11 11 30	0 12 17
5	9 14 48	10 16 19	11 15 30	0 16 13
9	9 18 53	10 20 22	11 19 30	0 20 8
13	9 22 57	10 24 25	11 23 29	0 24 3
17	9 27 1	10 28 27	11 27 28	0 27 57
21	10 1 5	11 2 28	0 1 26	1 1 51
25	10 5 10	11 6 30	0 5 23	1 5 44
29	10 9 13	11 10 30	0 9 20	1 9 37

Day	May S. ° ′	June S. ° ′	July S. ° ′	Auguſt S. ° ′
1	1 11 33	2 11 23	3 10 1	4 9 37
5	1 15 25	2 15 12	3 13 50	4 13 27
9	1 19 17	2 19 2	3 17 39	4 17 17
13	1 23 8	2 22 51	3 21 27	4 21 8
17	1 27 0	2 26 40	3 25 16	4 24 58
21	2 0 50	3 0 21	3 29 6	4 28 50
25	2 4 40	3 4 18	4 2 55	5 2 42
29	2 8 30	3 8 7	4 6 44	5 6 34

Day	September S. ° ′	October S. ° ′	November S. ° ′	December S. ° ′
1	5 9 28	6 8 47	7 9 37	8 3 55
5	5 13 21	6 12 44	7 13 38	8 13 59
9	5 17 14	6 16 41	7 17 40	8 18 3
13	5 21 8	6 20 39	7 21 41	8 22 7
17	5 25 2	6 24 38	7 25 44	8 16 11
21	5 28 58	6 28 37	7 29 46	9 0 16
25	6 2 53	7 2 37	8 3 49	9 4 21
29	6 6 49	7 6 36	8 7 53	9 8 26

By this table, and what hath been before delivered, it will be eaſy to find the declination for any one of the fore-mentioned

mentioned years, after the Biſſextile year, and for any day in that year, to almoſt any accuracy required.

485. The meaſure of the moon's horizontal diameter being generally between the limits of 29′ 20″ and 34′ 0″, the medium may be ſuppoſed 32′ nearly; and hence the augmentation of the moon's diameter by her elevation above the horizon for her leaſt, mean, and greateſt horizontal diameters will be nearly as in the following table.

Alt.	Leaſt	Mean	Greateſt	Alt.	Leaſt	Mean	Greateſt
°	″	″	″	°	″	″	″
5	2	3	3	50	21	25	29
10	5	6	7	55	23	27	31
15	7	9	10	60	24	29	32
20	10	11	13	65	25	30	34
25	12	14	16	70	26	31	35
30	14	17	19	75	27	32	36
35	16	19	21	80	27	33	37
40	18	21	24	85	28	33	37
45	20	23	26	90	28	33	37

486. The refractions have been reckoned nearly as follows.

	For London.		Equinoctial.	The Cape of Good Hope.
Altitude.	Summer.	Winter.		
°	′ ″	′ ″	′ ″	
0	31 0	35 0	27 0	
1	21 50	24 30	20 30	
	16 30	18 30	15 50	
3	12 50	14 30	12 25	
4	10 25	11 45	10 5	
5	8 40	9 45	8 20	
6	7 25	8 20	7 5	8 28
7	6 25	7 15	6 5	7 28
8	5 40	6 25	5 20	6 37
9	5 5	5 45	4 50	5 54
10	4 35	5 10	4 20	5 20
11	4 10	4 45	3 55	4 50
12	3 50	4 20	3 30	4 25
13	3 35	4 0	3 15	4 5
14	3 20	3 45	3 0	3 50
15	3 5	3 30	2 48	3 37
16	2 50	3 15	2 35	3 25

For

	For London.		Equinoctial.	The Cape.
Altitude.	Summer.	Winter.		
°	′ ″	′ ″	′ ″	′ ″
17	2 42	3 5	2 25	3 15
18	2 33	2 52	2 17	3 8
19	2 24	2 42	2 10	3 0
20	2 17	2 33	2 3	2 52
21	2 10	2 25	1 57	2 45
22	2 2	2 18	1 50	2 37
23	1 57	2 11	1 45	2 30
24	1 50	2 5	1 40	2 23
25	1 46	2 0	1 35	2 17
26	1 42	1 54	1 30	2 12
27	1 38	1 50	1 27	2 6
28	1 33	1 45	1 25	2 1
29	1 30	1 40	1 20	1 56
30	1 26	1 36	1 16	1 52

Here is a difference of 10″ between the ſummer and winter refraction for London.

Alt.	Lond.	Equin.	Cape.	Alt.	Lond.	Equin.	Cape.
°	′ ″	′ ″	′ ″	°	′ ″	′ ″	′ ″
31	1 28	1 14	1 47	46	0 52	0 42	1 3
32	1 25	1 10	1 43	47	51	41	1 0
33	1 20	1 8	1 39	48	49	40	58
34	1 18	1 6	1 36	49	47	39	56
35	1 15	1 4	1 32	50	46	38	54
36	1 13	1 2	1 29	51	44	35	53
37	1 10	1 0	1 26	52	42	33	51
38	1 8	0 58	1 23	53	41	32	49
39	1 5	0 55	1 20	54	39	31	47
40	1 3	0 53	1 17	55	38	30	45
41	1 1	0 51	1 14	56	37	29	44
42	0 59	0 49	1 12	57	35	28	42
43	0 57	0 48	1 10	58	34	27	41
44	0 55	0 46	1 7	59	33	25	39
45	0 53	0 44	1 5	60	31	25	38

Here is a difference of 6″ at the leaſt, and 13″ at the moſt, in the refractions at the three places, being compared with

with each other, as high as the altitude of 60°. This is applicable to what has been before delivered concerning the obliquity of the ecliptic, as derived from obſervations made at different places. But carrying the refractions from 60° to the zenith, leſs differences ariſe, and they are nearly as follows for the temperate zone.

Altitude.	Refraction.	Altitude.	Refraction.	Altitude.	Refraction.
°	′ ″	°	′ ″	°	′ ″
61	0 30	70	0 20	80	0 10
62	29	71	19	81	9
63	28	72	18	82	8
64	27	73	17	83	7
65	25	74	16	84	6
66	24	75	15	85	5
67	23	76	14	86	4
68	22	77	13	87	3
69	21	78	12	88	2
		79	11	89	1

487. The dip of horizon is uſually allowed for nearly as follows.

Height.	Dip.	Height.	Dip.	Height.	Dip.
Feet	′ ″	Feet	′ ″	Feet	′ ″
1	1 8	12	3 55	32	6 25
2	1 36	14	4 15	34	6 38
3	1 58	16	4 33	36	6 50
4	2 15	18	4 40	38	7 0
5	2 33	20	5 5	40	7 10
6	2 48	22	5 20	42	7 20
7	3 0	24	5 35	44	7 33
8	3 13	26	5 48	46	7 42
9	3 25	28	6 0	48	7 52
10	3 35	30	6 13	50	8 2

488. A Ta-

488. A TABLE, ſhewing the ſouthing of the ſun by mean ſolar time, for every eighth day of a Biſſextile year.

Day	January h ′ ″	February h ′ ″	March h ′ ″	April h ′ ″
1	12 3 59	12 14 2	12 12 31	12 3 44
9	12 7 33	12 14 40	12 10 38	12 1 23
17	12 10 32	12 14 26	12 8 22	11 59 19
25	12 12 46	12 13 27	12 5 54	11 57 40
29	12 13 34	12 12 43	12 4 39	11 57 3

Day	May h ′ ″	June h ′ ″	July h ′ ″	Auguſt h ′ ″
1	11 56 47	11 57 26	12 3 25	12 5 50
9	11 56 6	11 58 50	12 4 46	12 5 0
17	11 56 1	12 0 29	12 5 41	12 3 34
25	11 56 32	12 2 12	12 6 2	12 1 37
29	11 57 0	12 3 1	12 5 59	12 0 28

Day	September h ′ ″	October h ′ ″	November h ′ ″	December h ′ ″
1	11 59 33	11 49 26	11 43 47	11 49 42
9	11 56 55	11 47 7	11 44 9	11 53 6
17	11 54 8	11 45 18	11 45 25	11 56 56
25	11 51 22	11 44 9	11 47 34	12 0 56
29	11 50 3	11 43 52	11 48 57	12 0 54

This table will give the numbers for every intermediate day by halfing, and it may be made ſerviceable for the firſt, ſecond, and third years after Biſſextile year; but it will be readier to uſe a table for every fourth year.

The application of the more curious inſtruments is reſerved for another volume.

THE END.